Part 4.2: Elementary acoustic loads of a loudspeaker

Bruno Gazengel and Philippe Herzog

Table des matières

Introduction

I

A. Objective

Objective of this part is to present usual acoustic loads existing in the case of loudspeakers operating at low frequencies.

1. Requirements

- Electrical, mechanical and acoustic systems (section 2)
- Electromechanical and mechanoacoustic couplings (section 3)

B. Exercice : QCM(1)

[*Solution n°1 p 36*]

At low frequencies, loudspeaker is:

- ☐ First order resonant system
- ☐ Two degree of freedom system
- ☐ Second order resonant system
- ☐ One degree of freedom system

C. Exercice : QCM(2)

[*Solution n°2 p 36*]

At low frequencies, small enclosed volume (sealed enclosure) is:

- ☐ Acoustic mass
- ☐ Acoustic compliance
- ☐ Second order resonant system
- ☐ Acoustic resistance

D. Exercice : QCM(3)

[*Solution n°3 p 36*]

At LF, a bottle is:

- ☐ Acoustic mass
- ☐ Acoustic compliance
- ☐ Second order resonant system
- ☐ Acoustic resistance

E. Exercice : QCM(4)

[*Solution n°4 p 36*]

Interface between vibrating piston and ambient space:

- ☐ Preserves the mass (does not modify the mass of a piston)
- ☐ Increases piston mass
- ☐ Reduces piston mass
- ☐ Increases the apparent area of a piston

Definitions

II

What is an acoustic load?

Vbration of a membrane is influenced by presence of a fluid in contact with its faces.

For a slow movement, the fluid is driven by the diaphragm and moves with it, unless near walls opposing to this movement. If the vibration is very rapid, the inertia of the fluid prevents it from perfectly following the motion of the membrane, and it also undergoes a compression.

The term "acoustic load" refers to the reaction exerted by the fluid on the membrane, whose behavior is so modified.

The acoustic loads studied later in this course are:

- acoustic radiation (effect of the air in front of a vibrating piston mounted on a screen),
- sealed enclosure (sealed volume where the fluid is compressed),
- vented enclosure (Helmholtz resonator).

Acoustic radiation

III

A. Acoustic radiation

1. General principles

The most "useful" acoustic load of a loudspeaker is its radiation, that is to say a coupling between its membrane and ambient air, represented by an impedance Z_{ar} provided by the fluid to the membrane. This coupling leads to two phenomena:

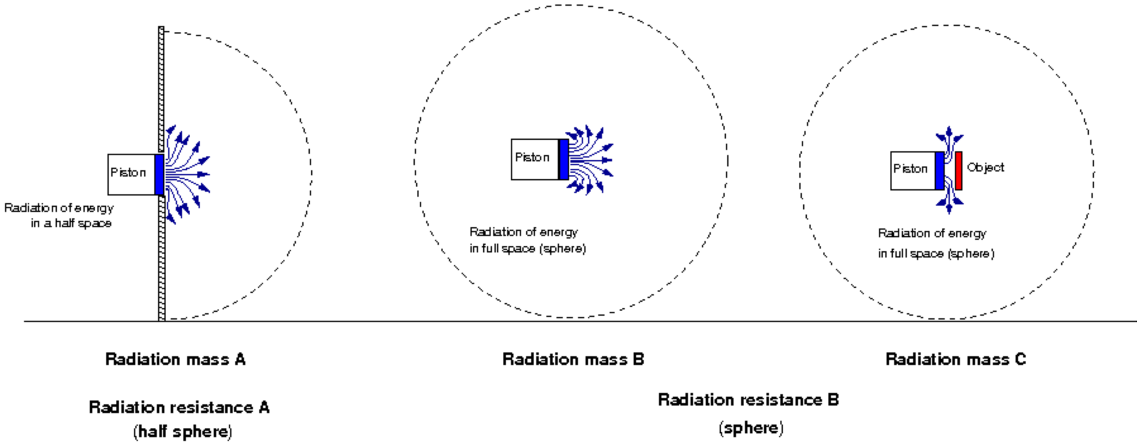
- At long distances (known as "far field"), pressure is a result of acoustic propagation between the loudspeaker and the point of listening. This propagation takes place only if a work is provided by the membrane to the fluid, so if it is compressed at the moment when the membrane is moving: reaction of the fluid must then be in phase with the vibration of the membrane , which corresponds to the real part of Z_{ar} .
- At very short distances (known as "near field"),movement of the fluid follows that of the membrane. Inertial force then largely dominates, and the reaction on the fluid corresponds essentially to the mass of the fluid thus driven, called "radiation mass". This corresponds to the imaginary part of Z_{ar} .

2. Examples

Exemple

The figure below illustrates three cases:

- Case A: radiation mass is determined by a presence of a screen in the "continuity" of an oscillating piston. Radiation resistance is due to the dispersion of the energy into a half-sphere.
- Case B: radiation mass is determined by the absence of a screen in the "continuity" of the oscillating piston. Radiation resistance is due to the dispersion of the energy into a sphere.
- Case C: radiation mass is determined by a presence of an obstacle in front of a piston and by the absence of screen in the continuity of an oscillating piston. The radiation resistance is the same as that of case B (dispersion into a sphere).



B. Radiation impedance

1. Radiation mass

Transition between piston velocity field (normal piston velocity) and sound field in free space is illustrated below:

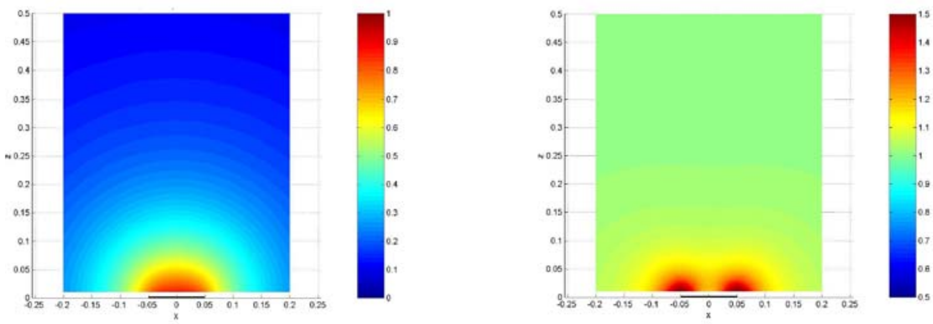


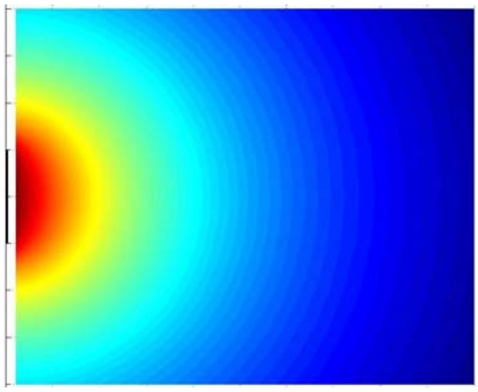
Figure on the left represents a sine of a phase shift ϕ between pressure and velocity. Pressure is in quadrature ($sin\phi = 1$) in the immediate vicinity of the membrane (radiation mass), and then tends to be in phase ($sin\phi = 0$) at greater distance (propagation).

Figure on the right represents a ratio between local acoustic velocity and that corresponding to a "pure" propagation situation. Additional (tangential) component dominates at the edges, as the fluid particles are "chased" outward rather than compressed (radiation mass);

2. Radiation impedance

Radiation impedance

The figure below shows a distribution of an active acoustic intencity in the vicinity of a vibrating piston. It is very high (red) on its surface, with a decay going from the center towards the edges (because of the tangential velocity at the edge). At a greater distance, the intensity decreases according to a profile which rapidly becomes spherical, and decreases with the distance tending towards zero (in blue).



Radiated power is conserved during propagation. At long distance ("far" field), the spatial dependence of the field tends to a sphere of radius *r* ;

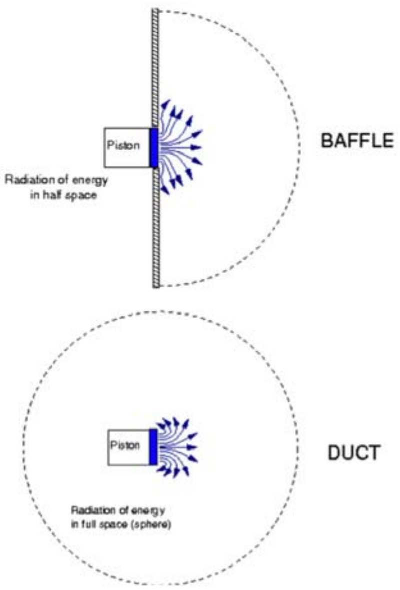
the acoustic intensity is then proportional to $\frac{1}{r^2}$ (*r* being the distance between source and receiver)

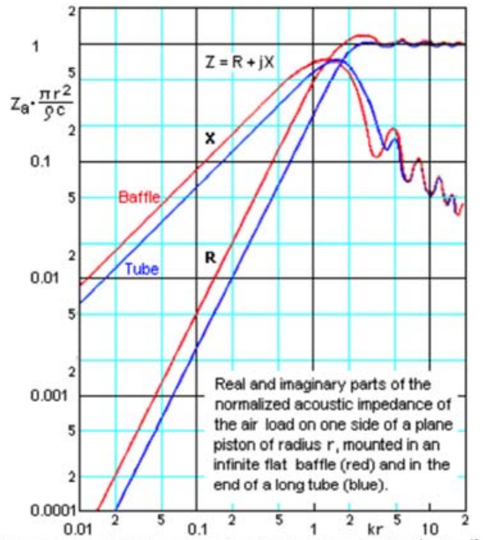
and the sound pressure is proportional to $\frac{1}{r}$;

3. Typical examples

Typical examples

Radiation impedance of a piston with a circular cross-section mounted in a baffle and that at the end of a tube are shown below:





Frequency is plotted on a normalized scale, where kr=2πfr/c=2πr/λ.
After Beranek: Acoustics, McGraw-Hill 1954, pp 119, 122.

In both cases, imaginary part (radiation mass) dominates at low frequencies, whereas at higher frequencies the real part dominates, tending towards the characteristic impedance (normal velocity component is dominant).

4. Approximated expression of a radiation impedance

At low frequencies (when the wavelength is large compared to the radius of the piston - $ka << 1$), the radiation impedance can be written in general terms:

$$Z_{ar} = Z_c[\alpha(ka)^2 + j\beta(ka)] = R_{ar} + j\omega M_{ar},$$

where $Zc = \frac{\rho c}{S}c$ is the characteristic impedance in pressure/volume velocity analogy. ρ is density of the air (1,2 kg/m^3 at 20 deg C), c is the speed of sound in air at rest ($c \approx 344 m/s$ at 20 deg C) and $S = \pi a^2$ is a section of a piston. k is a wave number $k = \frac{\omega}{c}$.

Resistance and radiation mass can thus be written:

$$R_{ar} = Z_c\alpha(ka)^2$$

$$M_{ar} = Z_c\frac{\beta a}{c} = \beta\frac{\rho a}{S},$$

where α depends on the radiation conditions (sphere, half-sphere, ...) and β on the mounting conditions of a piston.

5. Examples of radiation resistance and radiation mass

At low frequencies, two approximated expressions of resistances and radiation mass are often used:

	Radiation resistance	Radiation mass	Length correction
In a screen	$R_a = Z_c\frac{(ka)^2}{2}$	$M_a = \frac{8}{3\pi}\frac{\rho a}{S}$	$\Delta l = \frac{8}{3\pi}a \approx 0.86a$
In the end of a tube	$R_a = Z_c\frac{(ka)^2}{4}$	$M_a = 0.6133\frac{\rho a}{S}$	$\Delta l = 0.6133a$

Radiation mass is sometimes assimilated to that of a cylinder of fluid with same cross-section as the piston / guide, and then expressed as the length of this cylinder ("length correction").

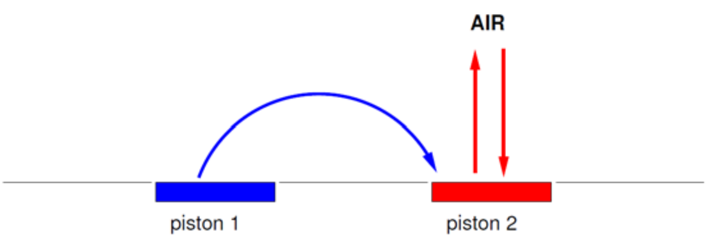
Remarque : Remark

At low frequencies $ka << 1$ so $(ka)^2 < ka$. The value of a radiation resistance R_{ar} remains small compared to the radiation reactance $X_{ar} = \omega M_{ar}$. The load effect on an acoustic source can thus be approximated by $Z_{ar} = j\omega M_{ar}$.

6. Mutual radiation impedance

Complément : Complement

In the case when several vibrating surfaces exist, each surface "sees" simultaneously the reaction of the medium (air) to its own vibration and the pressure generated by its neighbors (mutual influence).



In this case, the radiation impedance is no longer a scalar but a matrix:

$$\begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \cdot \begin{pmatrix} W_1 \\ W_2 \end{pmatrix}$$

The non-diagonal terms ("mutual" impedances) are essential in the case of several sources arranged close one to another, such as, for example, the speakers network (line array) or a vented enclosure (proximity to loudspeaker and vent).

C. Radiated pressure and directivity

1. Pressure radiated by a single source

Acoustic power which is transmitted by a source to the medium is written:

$$\Pi_a = R_{ar} w_{rms}^2$$

where R_{ar} is a real part of radiation impedance and w_{rms} is an acoustic output from the source. This power is propagated step by step, "to infinity".

For a monopolar source, pressure field is written:

$$p(r) = j\omega\rho w \frac{e^{-jkr}}{\Omega r}$$

where w is the volume velocity of the source and Ω is a solid angle in which the source radiates.

($\Omega = 4\pi$: infinite space, $\Omega = 2\pi$: half-space, etc).

Remarque : Remark

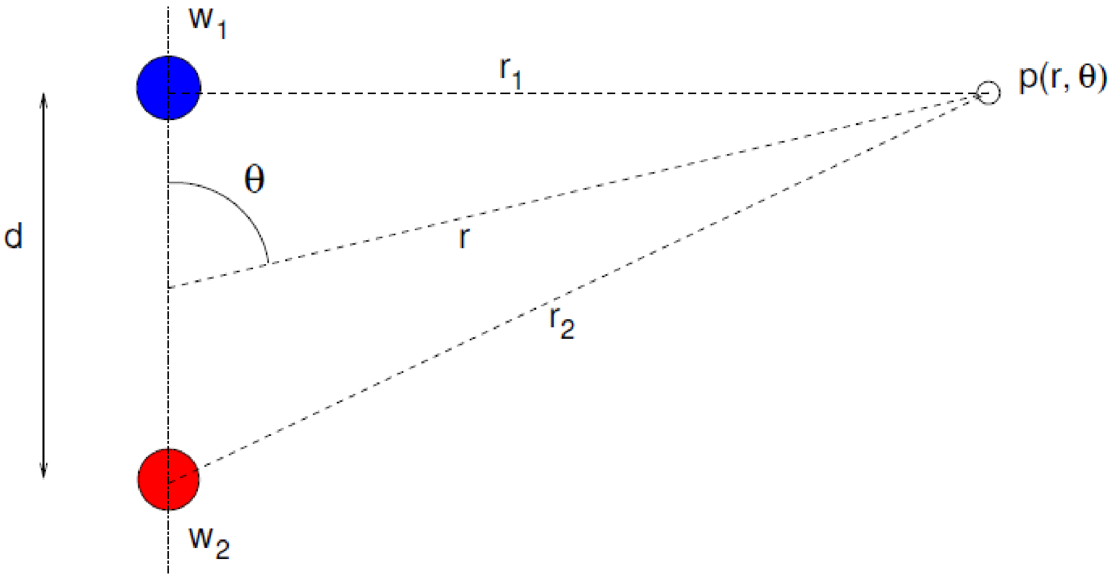
Equation 1 shows that:

- pressure amplitude decreases with $\frac{1}{r}$, which leads to the existence of spherical waves,
- it is proportional to the acceleration a_s and to the surface S of the source ($j\omega\rho w = \rho a_s S$),
- wave traveling time is $\tau = \frac{r}{c}$ (phase $\phi = -kr = -\omega\tau$)

2. Pressure radiated by two sources

In the case where two monopoles radiate in an infinite space, with two volume velocities w_1 and w_2, the pressure is a sum of two contributions and it is written as:

$$p_t(r,\theta) = p_1(r) + p_2(r) = j\omega\rho(w_1 \frac{e^{-jkr_1}}{4\pi r_1} + w_2 \frac{e^{-jkr_2}}{4\pi r_2}).$$



At large distance $(\{r_1, r_2\}) >> d)$, it leads to $r_1 \approx r_2 \approx r = R$ so

$$p_t(r,\theta) = \frac{j\omega\rho}{4\pi R}(w_1 e^{-jkr_1} + w_2 e^{-jkr_2}) = jk\rho c \frac{e^{-jkR}}{4\pi R} D(\theta,\Phi)$$

where $D(\theta,\Phi)$ characterizes the directivity of two-source system.

3. Directivity of a two-source system

Directivity term is expressed by writing $r_1 = r - \Delta r$ and $r_2 = r + \Delta r$ and assimilates pair of sources to a single one centered in the middle ("far field" approximation). It is written:

$$D(\theta,\Phi) = w_1 e^{jk\Delta r} + w_2 e^{-jk\Delta r}$$

$D(\theta,\Phi) \approx (w_1 + w_2) + (w_1 - w_2) jk\Delta r$ (if $k\Delta r << 1$)

Remarque : Remark

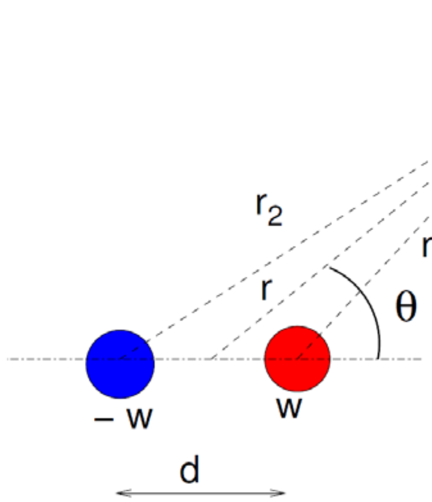
This equation shows that directivity depends

- on the ratio between source spacing *d* and the wavelength ($k\Delta r = k\frac{d}{2}cos\theta$).
- on the amplitude and the relative phase of two sources, w_1 and w_2 being complex numbers that represents amplitude and phase differences

The dependence of term $D(\theta,\Phi)$ on terms w_1 and w_2 allows to choose the directivity (and frequency response) for particular applications.

4. Directivity of a dipole

Considering a system of two monopol sources spaced at a distance *d* and generating volume velocities $w_1 = w$ and $w_2 = -w$. The size of the sources and the spacing between them are assumed to be small comparing to the wavelength.



Far field

It is shown that the pressure response in a dipole plane in far field is written :

$$\frac{p_d(r,\theta)}{p_m(r)} \approx 2jk\Delta r = jkdcos\theta = j\omega\frac{dcos\theta}{c}$$

where $p_m(r)$ is a pressure generated by a monopol of volume velocity w :

$$p_m(r) = j\omega\rho w\frac{e^{-jkr}}{4\pi r}$$

Near field

On the other hand in near field , we have :

$$\frac{p_d(r,\theta)}{p_m(r)} \approx \frac{jkdcos\theta}{jkr} = \frac{dcos\theta}{r}$$

The expression is proportional to ω in far field, but not in near field: this is what is called "proximity effect" (greater presence of low frequencies for a source close to the microphone).

5. Field radiated by a dipole

The dipole's directivity pattern is shown at the figure below. It is bidirectional:
$(D(\theta) = cos\theta)$

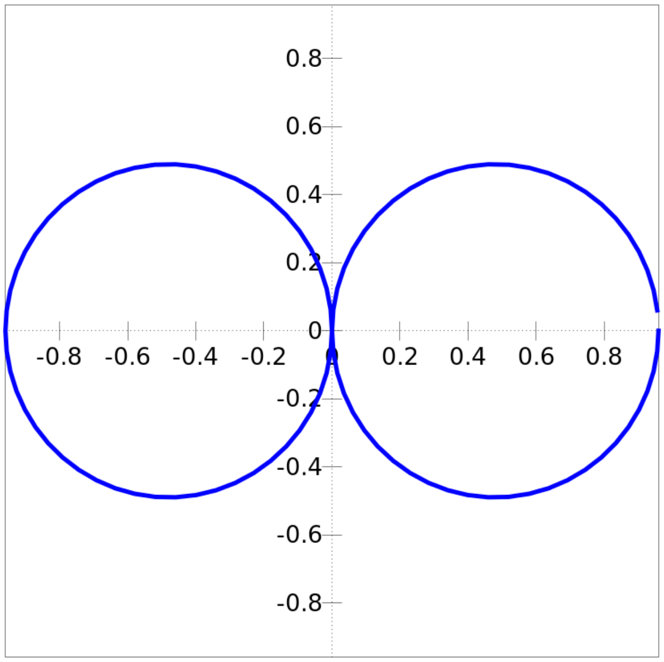
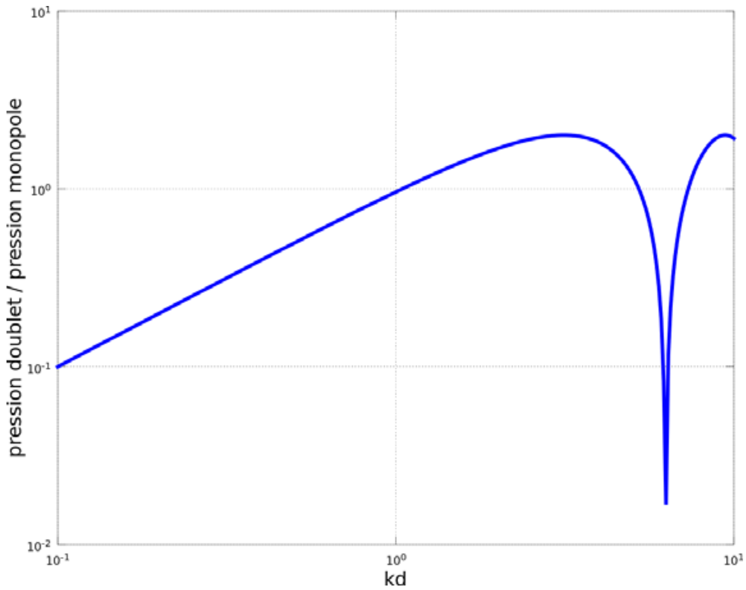


Diagramme de directivité du dipôle

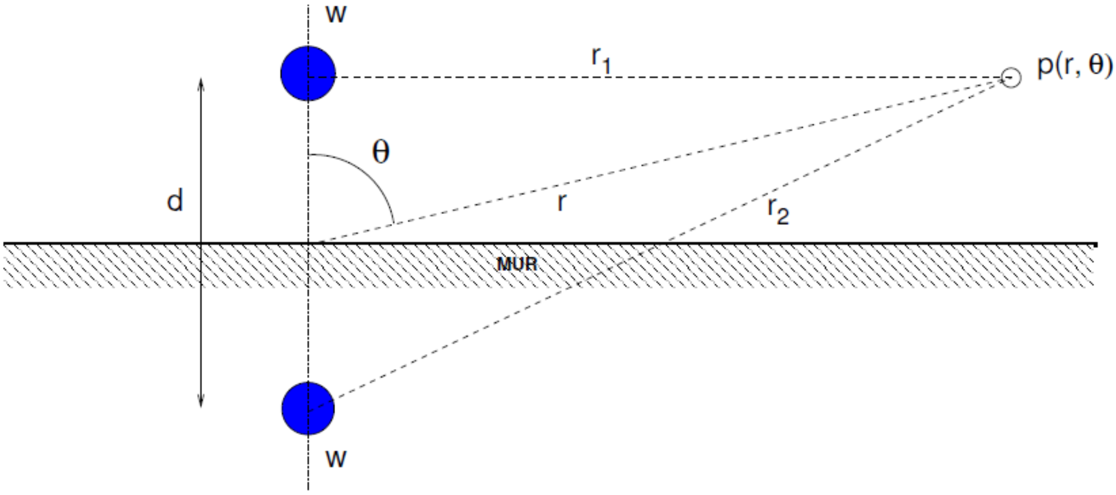
- In the far field: the dipole generates a first order high-pass spectrum comparing to the monopole (see figure below).
- In the near field: the dipole generates a spectrum identical to that of the monopole.



Pressure response of the dipole comparing to the monopole response for kd = 1.

6. Directivity of a source close to a wall

Considering a monopolar source (enclosure of small dimensions compating to the wavelength) placed close to a perfectly reflective infinite surface.

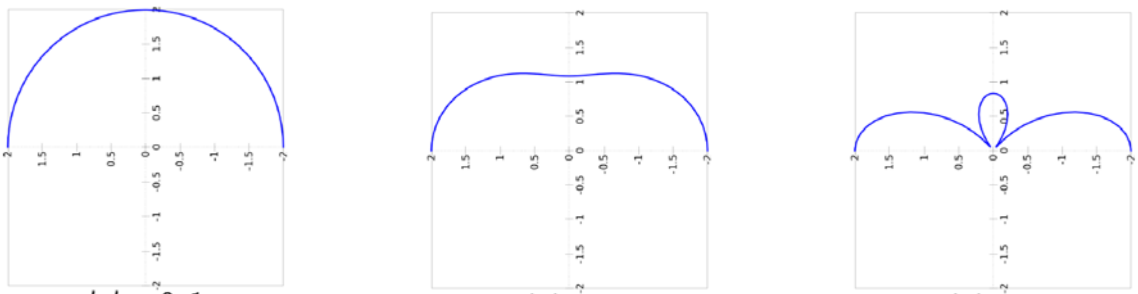


Effect of the wall can be seen as an existence of an image source producing a volume velocity identical to the actual one.

The pressure response of such a system is written in the far field and at low frequencies (kd << 1) as:

$$\frac{p(r)}{p_m(r)} = 2cos(\frac{kd}{2}cos\theta) \approx 2 - \frac{(kd)^2}{2}cos^2\theta$$ (avec $p_m(r) = j\omega\rho w \frac{e^{-jkr}}{4\pi r}$).

Directivity diagrams of such system are presented below for three values of *kd*.

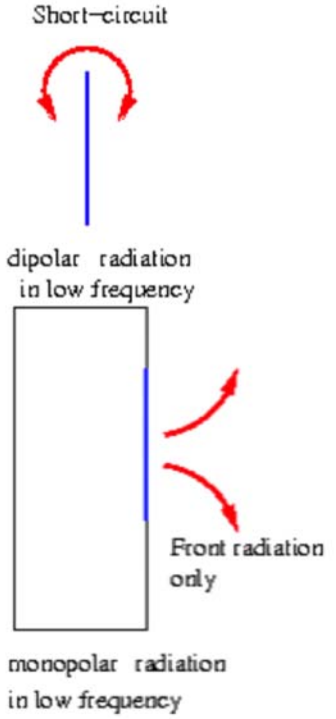

$kd = 0,1$ $kd = 2$ $kd = 4$

The main effect of the wall is to double the pressure (which corresponds to redused solid radiation angle), but the finite distance between the sources also induces a slight directivity, which becomes important only for $kd \geq 1$.

Sealed enclosure

IV

A. Principle



A vibrating surface (such as a membrane of a loudspeaker) radiates through its both faces, with a phase in front opposite to the rear. At low frequencies, this surface can be seen as an equivalent dipole and the resulted radiation is very inefficient (acoustic short circuit).

A simple way to improve that, is to isolate one side by closing it with a rigid enclosure. Thus, only open surface will radiate.

B. Enclosure response at low frequencies

Fondamental : Hypothesis

We assume here that the largest dimension *L* of the enclosure is small comparing to the wavelength. This can be written as *kL << 1* (where *k* is a wave number) or (where *V* is an enclosure volume).

In practice: For "column" enclosures for which the largest dimension may be greater than the wavelength, it is often necessary to consider a waveguide element along its length. The reasoning remains valid for the lowest frequencies.

Movement of a vibrating surface changes the volume inside the enclosure, which reacts by a pressure variation. At low frequencies, the air inside can be approximated to a spring (flexibility of the air inside a closed volume, part 2.3.2.).

In the vicinity of the membrane, the velocity distribution leads to a discontinuity mass, which depends on geometry of a problem (part 2.3.2.).

C. Loudspeaker, enclosure coupling

1. Coupling with a loudspeaker

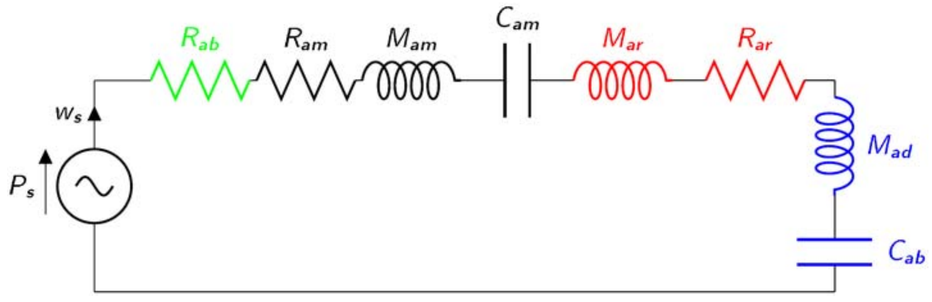
The coupling between loudspeaker and enclosure lead to the presence of:

- rear load impedance, comprising:
  - discontinuity impedance (mass) reflecting the geometrical rupture between loudspeaker membrane and volume of the enclosure $Z_{ad} = j\omega M_{ad}$
  - impedance linked to the compressibility of a fluid inside an enclosure, expressed by a compliance: $Z_{ab} = \dfrac{1}{j\omega C_{ab}}$
- radiation impedance of a front surface $Z_{ar} = R_{ar} + j\omega M_{ar}$

2. Acoustic equivalent circuit

An equivalent circuit of a speaker mounted in a sealed enclosure **in acoustic domain** is presented below with the following color conventions:

- Green: electrical part
- Black: mechanical part
- Red: radiation impedance (front surface)
- Blue: sealed enclosure (rear surface)



Remarque : Remark

For a "fairly large" enclosure, the discontinuity masses in the front and in the back of a membrane are very close $M_{ad} \approx M_{ar}$.

3. Equivalent acoustic parameters

Equivalent parameters given at the previous figure **in acoustic domain** can be expressed according to given physical parameters as follows:

Element	Electrical parameters	Mechanical parameters	Acoustic parameters
Resistance	$R_{ab} = (\frac{Bl}{Sd})^2 \frac{1}{R_{eb}}$	$R_{am} = \frac{R_{mm}}{S_d^2}$	$R_{ar} = Z_c \frac{1}{4}(kr_d)^2$

Inductance		$M_{am} = \frac{M_{mm}}{S_d^2}$	$M_{ar} = \rho\frac{\alpha r_d}{S_d}$, $M_{ad} = \rho\frac{\beta r_d}{S_d}$
Capacitance		$C_{am} = C_{mm}S_d^2$	$C_{ab} = \frac{V_b}{\rho c^2}$

where ρ and c are respectively the air density and the speed of sound in the air, $k = \frac{\omega}{c}$ is a wave number, α and β are the coefficients characterizing an effect of discontinuity between the membrane and the air. In the case of a membrane in infinite screen $\alpha = \frac{8}{3\pi} \approx 0.85$ and for a membrane in infinite space $\alpha \approx 0.6133$. Besides, the source pressure is written as $P_s = \frac{BlU_s}{S_d R_{eb}}$.

4. Radiated pressure

Acoustic pressure radiated by a speaker (supposed to be placed in infinite space and small comparing to the wavelength) is written:

$$p(r) = j\omega\rho w_s \frac{e^{-jkr}}{4\pi r}.$$

Speaker response, defined as a ratio between radiated pressure and supply voltage, is given by $\frac{p(r)}{U_s}$

and depends on the volume velocity response $\frac{w_s}{U_s}$ (ratio between volume velocity generated by the speaker and supply voltage).

This response therefore depends on the acceleration response of the membrane:

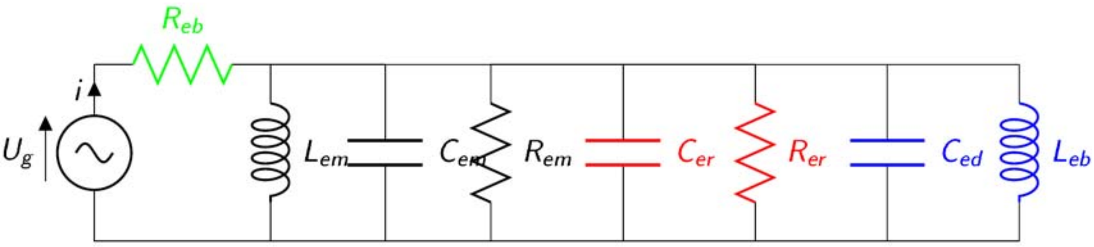
$$\frac{p(r)}{U_s} = \rho S_d \frac{a_s}{U_s}\frac{e^{-jkr}}{4\pi r}$$

where a_s is an acceleration in center of a membrane.

5. Electrical equivalent circuiit

An equivalent circuit of a speaker mounted in a sealed enclosure **in electrical domain** is presented below with the following color conventions (electrical inductance L_{eb} is neglected here):

- Green: electrical part
- Black: mechanical part
- Red: radiation impedance (front surface)
- Blue: sealed enclosure (rear surface)



6. Equivalent electrical parameters

Equivalent parameters given at the previous figure **in electrical domain** can be expressed according to given physical parameters as follows:

Element	Electrical parameters	Mechanical parameters	Acoustic parameters
Resistance	R_{eb}	$R_{em} = \frac{(Bl)^2}{R_{mm}}$	$R_{er} = (\frac{Bl}{S_d})^2 \frac{1}{R_{ar}}$
Inductance		$L_{em} = (Bl)^2 C_{mm}$	$L_{eb} = (\frac{Bl}{S_d})^2 C_{ab}$
Capacitance		$C_{em} = \frac{M_{mm}}{(Bl)^2}$	$C_{er} = (\frac{S_d}{Bl})^2 M_{ar}$, $C_{ed} = (\frac{S_d}{Bl})^2 M_{ad}$

7. Enclosure influence on the resonance frequency and quality factor

Acceleration response of the speaker can be calculated by analyzing presented equivalent scheme.

Volume velocity response is written $\frac{P_s}{w_s} = R_{eq} + j\omega M_{eq} + \frac{1}{j\omega C_{eq}}$, where $R_{eq} = R_{ab} + R_{am} + R_{ar}$ (electrical + mechanical + acoustic radiation losses), $M_{eq} = M_{am} + M_{ar} + M_{ad}$ (mechanical + acoustic radiation + discontinuity masses),

$C_{eq} = \frac{C_{ab} C_{am}}{C_{ab} + C_{am}}$.

If we compare this response to that of a screen-mounted loudspeaker, the term that differs is compliance C_{eq} (assuming that the masses and radiation resistances are identical in both cases). Thus, the addition of an enclosure stiffness to the loudspeaker modifies its resonance frequency as well as its quality factor.

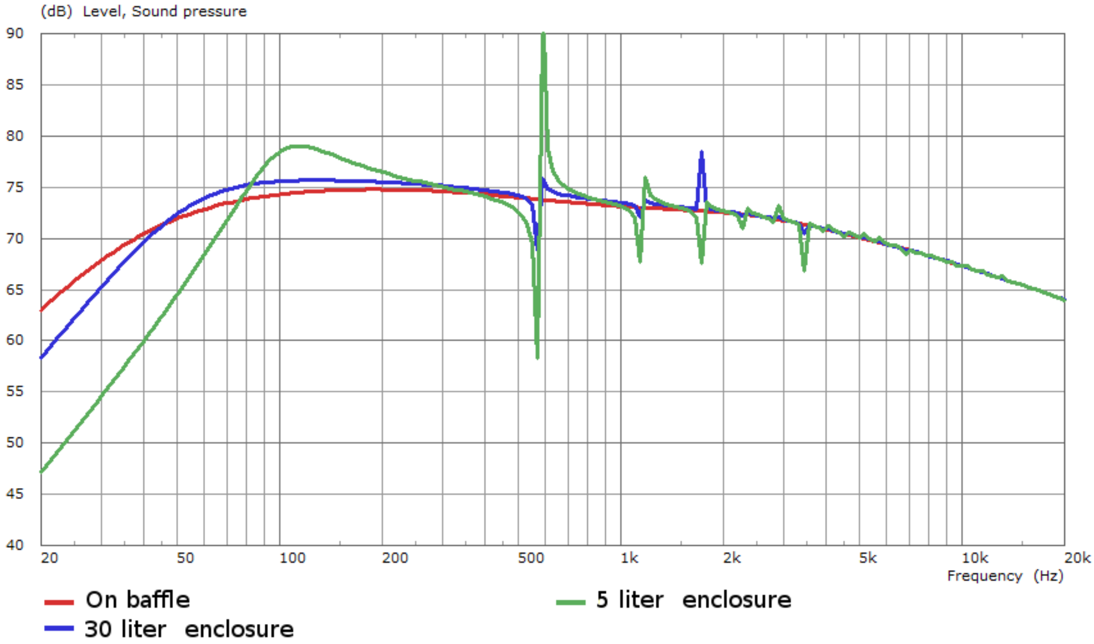
Table below gives the values of resonance frequencies and quality factors for the loudspeaker mounted in a screen and in a sealed enclosure.

System	Resonance frequency	Quality factor
Speaker in a screen	$f_s = \frac{1}{2\pi \sqrt{M_{eq} C_{am}}}$	$Q_{ts} = \frac{1}{\omega_s C_{as} R_{eq}}$
Speaker in a sealed enclosure	$f_b = f_s \sqrt{1+\alpha}$	$Q_{tb} = Q_{ts} \sqrt{1+\alpha}$

where $\alpha = \frac{C_{am}}{C_{ab}} = \frac{V_{as}}{V_{ab}}$.

8. Frequency response

Frequency response of a speaker's membrane acceleration ($\frac{a_s}{U_s}$) is presented at the figure below. It was obtained using Akabak software [2] and the loudspeaker whose characteristics are given in *annex 4*.



Frequency response is calculated for three different configurations:

- loudspeaker in an infinite screen
- loudspeaker mounted in the enclosure of 30 liters volume
- loudspeaker mounted in the enclosure of 5 liter volume

In the case of loudspeaker mounted in an infinite screen, its response has a high-pass type and it is very damped, which can be explained by the low value of total quality factor Q_{ts} (0,52).

In the case of loudspeaker mounted in 30 liter enclosure, resonance frequency of the assembly is higher than that of the loudspeaker in a screen. Total quality factor is also higher.

Finally, in the case of loudspeaker mounted in 5 liter enclosure, resonance frequency and total quality factor are clearly higher than in the case of the screen-mounted loudspeaker.

For both enclosures, the model takes into account the eigenmodes of the enclosure along its axis of greatest dimension, which translates into visible resonances on the pressure response (1st eigen resonance frequency is at about 550 Hz). Amplitude of these resonances is a priori overestimated by the software which does not take into account all losses inside the enclosure.

9. Electrical impedance

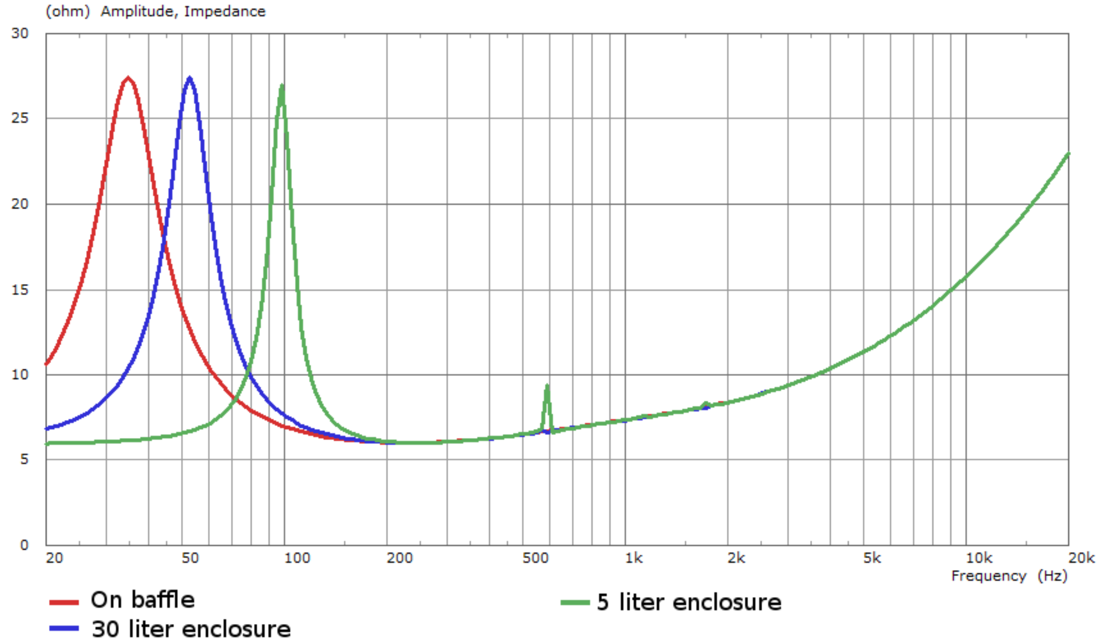
Electrical impedance, that was obtained with Akabak software [2] and using the loudspeaker whose characteristics are given in *annex 4*, is shown on the figure below :

This figure shows the electrical impedances calculated for the three cases:

- loudspeaker in an infinite screen
- loudspeaker mounted in the enclosure of 30 liters volume
- loudspeaker mounted in the enclosure of 5 liter volume

Effect of the enclosure is clearly visible and shows an increase of the resonance frequency due to the effect of the air inside the volume which increases the mechanical stiffness of the whole system.

Effect of the eigenmodes of a cavity on the impedance curve (at 550 Hz approximately) appears for the enclosure of 5 liter volume.


(ohm) Amplitude, Impedance
30
25
20
15
10
5
0
20
50
100
200
500
1k
2k
5k
10k
20k
Frequency (Hz)
On baffle
30 liter enclosure
5 liter enclosure

Vented enclosure

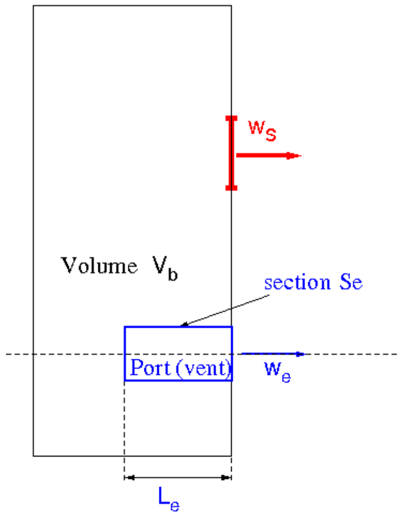
V

A. Principle

Vented enclosure is a system that uses a volume and a vent in a way, that entire rear load acts as a Helmholtz resonator.

Characteristics of a resonator are:

- Volume V_b,
- Length of a vent L_e ,
- Cross-section of the vent S_e .



By convention, the volume velocities are assumed to be positive when the fluid leaves the enclosure.

B. Enclosure response at low frequencies

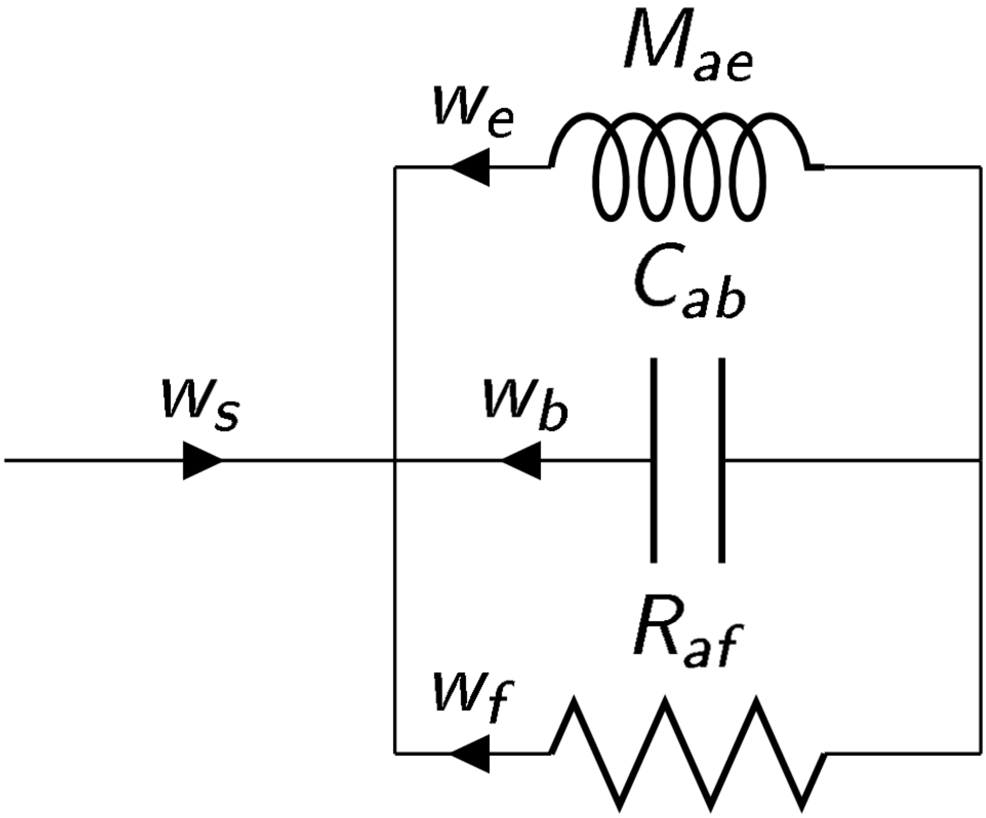
1. Effect of the enclosure (1)

Rear surface of a loudspeaker radiates into a resonant system which presents:

- compliance C_{ab} due to the compressibility of the air inside a volume,
- acoustic mass $M_{ae} = \dfrac{\rho(L_e + \Delta l)}{S_e}$ due to the inertia of the air in a vent. The term Δl represents a length correction due to the discontinuity effects at the input and at the ouput of a vent. Radiation resistance is neglected here.
- resistance R_{af} which represents the losses inside the enclosure (often neglected). This resistance determines a quality factor of the system Q_b (often between 5 and 10 for a neat enclosure).

2. Effect of the enclosure (2)

Rear load seen by a loudspeaker can be represented by the equivalent diagram below. Volume velocity which is required to compress or depress the air in a volume is opposite to the sum of volume velocities $w_b = -(w_s + w_e + w_f)$. Knowing that w_s ,w_e ,w_f are positively directed outside the enclosure, represents the fact that the air while leaving enclosure creates a negative volume velocity, that is a decompression of the air.



3. Acoustic impedance of the enclosure

Rear admittance seen by a loudspeaker (neglecting R_{af}) can thus be written as $Y_{ab} = j\omega C_{ab} + \frac{1}{j\omega M_{ae}}$, which leads to a rear impedance:

$$Z_{ab} = \frac{j\omega M_{ae}}{1-(\frac{\omega}{\omega_r})^2},$$

where $\omega_r = \frac{1}{M_{ae}C_{ab}}$.

Ratio between volume velocities is:

$$\frac{w_e}{w_s} = \frac{-1}{1-(\frac{\omega}{\omega_r})^2}.$$

Impedance and volume velocity analysis is given in the table below.

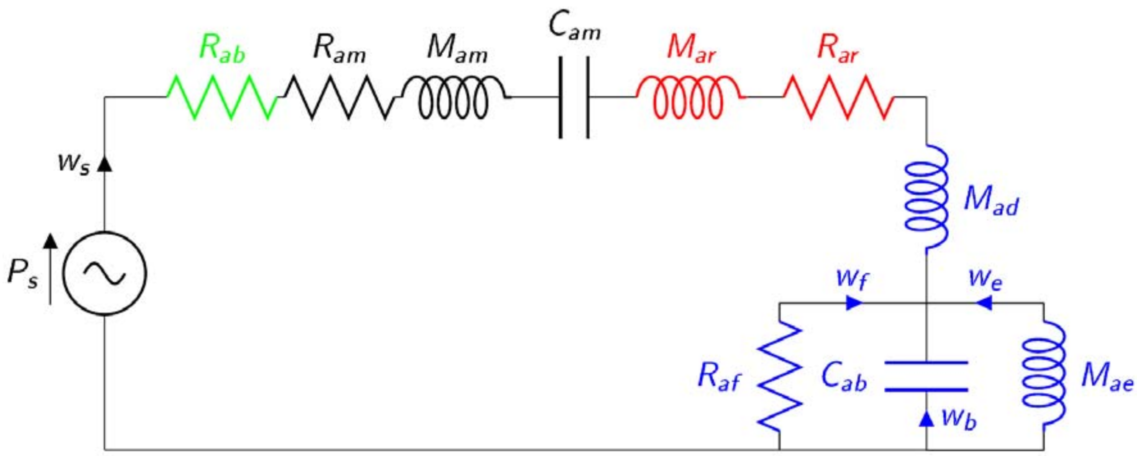
Frequency	$\omega < \omega_r$	$\omega \approx \omega_r$	$\omega > \omega_r$

Volume velocity ratio	$w_e = -w_s$	w_s maximum	$w_s \rightarrow 0$
Impedance seen by the speaker	$Z_{ab} \approx j\omega M_{ae}$	Z_{ab} maximum and resistive	$Z_{ab} \approx \frac{1}{j\omega C_{ab}}$
Effect of a resonator	Added mass	Volume velocity amplification	Sealed enclosure

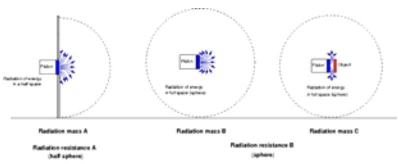
C. Loudspeaker + enclosure combination

1. Combination of both radiations

Combination of a loudspeaker and a Helmholtz resonator results in a two degrees of freedom system, which equivalent diagram in acoustic domain is shown below. Color conventions are identical to those presented in *acoustic equivalent circuit* .



2. Pressure radiated by a speaker

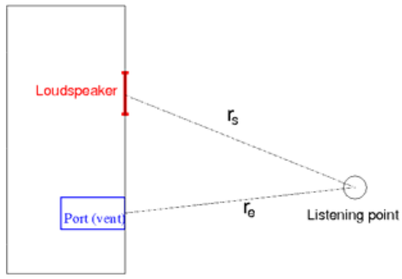


$$\frac{p(r)}{P_s} = jk\rho c(\frac{w_s}{P_s}\frac{e^{-jkr_s}}{4\pi r_s} + \frac{w_e}{P_s}\frac{e^{-jkr_e}}{4\pi r_e})$$

Assuming that the listening point is on the axis or in a far field, it comes to $r_e \approx r_s = r$ and the radiated pressure is directly proportional to $w_s + w_e$.

Thus, it is written as a function of the volume velocity w_b that is necessary to compress the air inside the volume

$$V_b = \frac{p(r)}{P_s} = j\omega\rho\frac{w_b}{P_s}\frac{e^{-jkr}}{4\pi r}.$$



3. Alignment

Coupling between two resonant systems of one degree of freedom (loudspeaker + Helmholtz resonator) creates a two degrees of freedom system with two resonances. In fact, the pressure response, proportional to the volume velocity response $\frac{w_b}{P_s}$, is a response of order 4 which is written in general terms as:

$$p(r) = \frac{U_g}{R_e}\frac{\rho BlS_d}{M_{ms}}\frac{e^{-jkr}}{4\pi r}G(s)$$

where $G(s) = \frac{s^4}{s^4 + P_3 s^3 + P_2 s^2 + P_1 s + P_0}$, noting $s = j\omega$.

Pressure response is therefore a high-pass filter of order 4 having a slope of 24 *dB/oct*.

So, it is necessary to choose the dimensions of a Helmholtz resonator in such a way that the volume velocity generated by the vent is neither too large nor too small to obtain a suitable response at low frequencies, that is to say not showing any too visible resonance, which caused dragging effects in the vicinity of the resonant frequency of the box.

4. Alignment (continuation)

For this purpose, the known forth order filter responses are used to identify the terms of a theoretical response of the speaker under the terms of the response of the filter. Thus, we can determine the "typical responces" of a vented enclosure to quickly determine a geometry of an enclosure depending on the speaker parameters.

It is usually considered that the quality factor of the speaker is $Q_b = 7$. For this case, some classical alignments are given below (additional values are available in Beranek [1])

Remarque : Remark

Some values that are used for the dimensioning of vented enclosures. f_s is the resonance frequency of the loudspeaker, f_{3dB} is the *3 dB* cutoff frequency of the loudspeaker, V_b is a volume of a loudspeaker, V_{as} is loudspeaker parameter associated with its mechanical flexibility , Q_{ts} is total quality factor of a loudspeaker, f_b is resonance frequency of a Helmholtz resonator.

Alignement	f_{3dB}/f_s	$V_b = V_{as}$	Q_{ts}	f_b/f_s
Sub-Butterworth	1.7748	0.4028	0.3010	1
Butterworth	1	0.9422	0.405	1
Chebyshev (0.01 dB ripple)	0.8143	1.5511	0.4572	0.8838
Chebyshev (0.25 dB	0.6374	2.9747	0.5553	0.7259

ripple)				

5. Calculation of the enclosure parameters

Dimensions of a vented enclosure may be determined by means of graphs or by numerical calculations.

Calculation with graphs

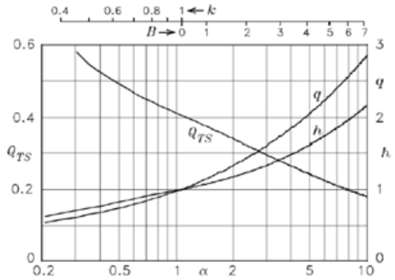


Image 1 Curves for determining the dimensions of a vent enclosure

- Use Figure 1 which is an extension of Table 1 [3].
- From Q_{ts} of a loudspeaker on the left axis. Join the curve Q_{ts} with a horizontal line.
- Deduce the value of α and calculate the volume of the enclosure $V_b = \frac{V_{as}}{\alpha}$.
- Deduce the value of $h = \frac{f_b}{f_s}$
  Calculate the resonant frequency of the enclosure $f_b = hf_s$
- Deduce the value of q. Calculate the cut-off frequency at *-3 dB* $f_{3dB} = qf_s$

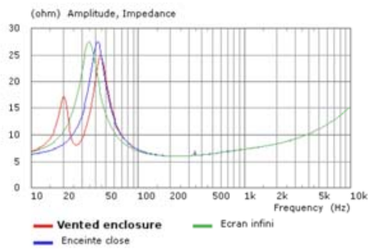
Numerical calculation

In this case, it is possible to use various speaker simulation software, such as those mentioned on this page:

http://www.speakerbuilding.com/software/[1]

6. Example of speaker calculation: electrical impedance

An example of calculation is given for the Visaton 170S loudspeaker. Volume of the enclosure considered here is 95 L and the dimensions of the vent are: length = 11 cm, diameter = 6 cm, which leads to the resonance frequency of a Helmholtz resonator of approximately 27.5 Hz. The electrical impedance is shown at the figure to the right.

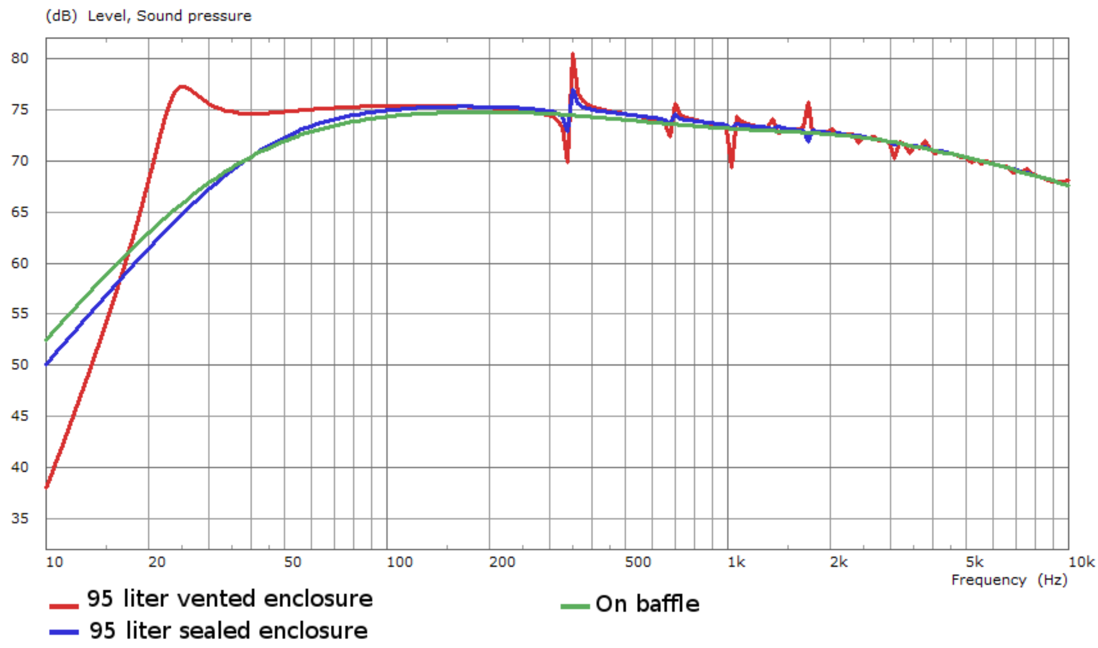


Impedance of the vented enclosure shows two peaks wich leads to the presence of two resonances, contrary to what happens with a screen-mounted speaker or sealed enclosure.

7. Example of speaker calculation: pressure response

Pressure response at 1 meter is shown at the figure below for the same loudspeaker (Visaton 170S, see *annex 4*) mounted in three different configurations. Enclosure has the same characteristics as above mentioned and its longest internal dimention is 50 cm.

1 - http://www.speakerbuilding.com/software/



8. Analysis of the simulation

Calculated results clearly shows the effect of the vented enclosure on the system response. In the case of sealed enclosure, the cut-off frequency at -3 dB is about 45 Hz and the slope is 12 dB / octave. For the vented enclosure, the cut-off frequency at -3 dB is about 25 Hz and the slope is 24 dB / octave.

In addition, some bursts occurring above 300 Hz are due to the eigen resonances of the box. The amplitude of these resonances is probably overestimated in the simulation.

9. Excursion of loudspeaker membrane

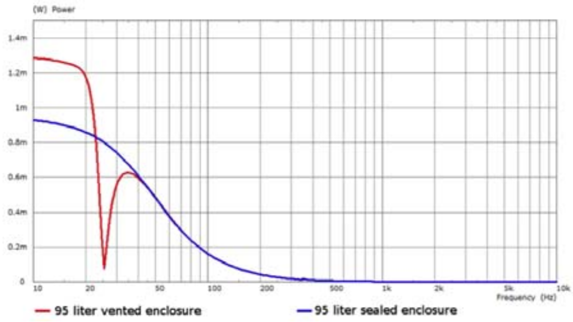
Complément : Complement

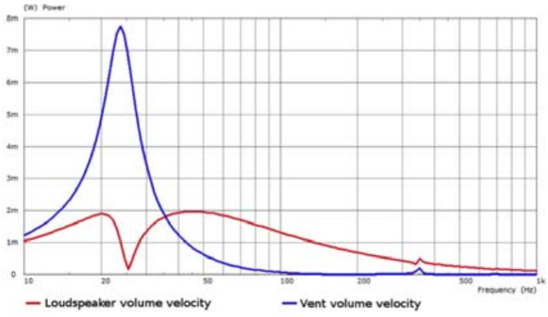
- Sealed enclosure: in the case of a sealed enclosure, an excursion of loudspeaker membrane remains constant below the resonance frequency. After, it decreases with a slope of 12 dB / oct.
- Vented enclosure : vented enclosure has an inertial type impedance at very low frequencies. Below the resonance frequency of Helmholtz resonator, loudspeaker must push the air mass on its rear surface. It then creates two flows in opposite phase (of the loudspeaker membrane and of the vent). At these frequencies, loudspeaker has a great range of excursion. It tends to minimum at a resonance frequency of Helmholtz resonator, for which the mechanical energy is transmitted optimally into the enclosure.

10. Excursion of loudspeaker membrane : example

An example of the excursion calculation is shown at the figure below on the left . It is done for the Visaton 170S loudspeaker mounted in the enclosure shown *previously*.

For the vented enclosure, volume velocities generated by the loudspeaker and the vent are shown at the figure below on the right. This figure clearly shows the role played by the vent which is to compensate a small displacement of the loudspeaker at the resonance frequency.



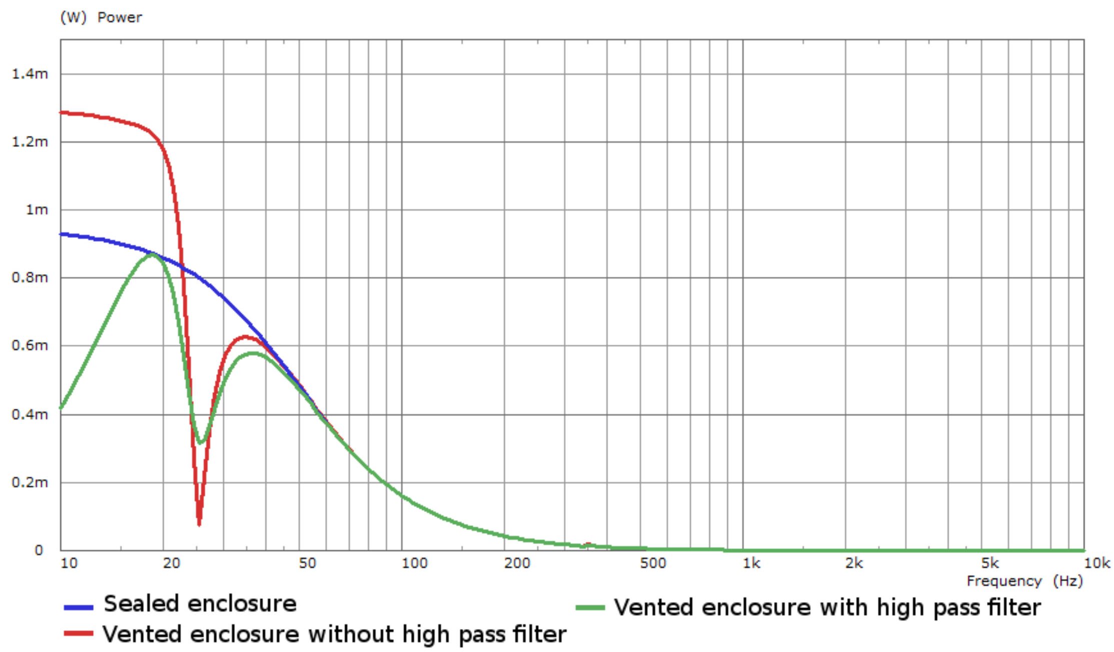


11. Active filtering of a vented enclosure

Complément : Complement

In order to limit an excursion of a loudspeaker membrane, it is common to use a high-pass filtering. Since the cut-off frequency of such a filter is low (20 to 30 Hz), it is preferable to use an active filter. Example of an excursion response is shown at the figure below for the sealed enclosure, unfiltered vented enclosure and vented enclosure provided with a Sallen & Key filter with 17 Hz cut-off frequency. This figure shows that by adding a filter (green curve) excursion is reduced between 10 and 20 Hz.

Figure below shows that the loudspeaker excursion for non filtered vented enclosure is very high below the resonance frequency of a Helmholtz resonator. If the loudspeaker is used for very low frequency signals, it may generate distortions and, on the other hand, may provide a mechanical damage which does not exist in a case of closed enclosure (excursion being lower). Application of a high-pass filter helps to limit the value of excursion at low frequencies and thus to limit the distortion effects and to protect the loudspeaker.



Conclusion

VI

A. Synthesis

1. Loudspeaker radiation

Loudspeaker which radiates in a given space (defined by its solid angle) exposed to an acoustic impedance, called a radiation impedance, which comprises at low frequencies (ka <1, where k is the wave number and a is the radius of loud speaker) :

- **Radiation resistance.** Radiation resistance depends on the far field radiation conditions (solid angle in which sound power is delivered). For a solid angle of 4π steradians (spherical radiation), radiation resistance is proportional to $\frac{(ka)^2}{4}$. For a solid angle of 2π (half space), radiation resistance is doubled and it is proportional to $\frac{(ka)^2}{2}$
- **Radiation mass.** Radiation mass depends on the geometry of the discontinuity between the loudspeaker and surrounding environment.

2. Loudspeaker in a sealed enclosure

Loudspeaker mounted in a sealed enclosure sees:

- on a front surface: radiation impedance
- on a rear surface: discontinuity impedance (acoustic mass) in series with an acoustic impedance of a box
    - at low frequencies, sealed enclosure is seen as acoustic compliance
    - at highest frequencies, the eigenmodes of the box reveal internal resonances modifying the impedance seen by the loudspeaker. In general, electroacoustic models take into account the resonances existing along the largest dimension of the enclosure using waveguide models (see section 2.3).

Response of a complete system (loudspeaker + sealed enclosure) is as follows:

- electrical impedance has a single resonance with frequency and quality factor that are higher than for a screen-mounted loudspeaker.
- pressure response is a high-pass filter of order 2 (12 dB / octave slope) with higher resonance frequency and quality factor than for screen-mounted loudspeaker. At high frequencies, pressure response may involve bursts due to the internal resonances of the box (which can be damped with absorbent material).

3. Loudspeaker in a vented enclosure

Vented enclosure is a cavity coupled with a tube. This assembly acting as a Helmholtz resonator in order to reinforce sound emission at low frequencies.

Loudspeaker mounted in a vented enclosure sees:

- on a front surface: radiation impedance
- on a rear surface: discontinuity impedance (acoustic mass) in series with acoustic impedance of a Helmholtz resonator
  - at low frequencies, vented enclosure is seen as a Helmholtz resonator
  - at higher frequencies, eigenmodes of the cavity reveal internal resonances modifying the impedance seen by the loudspeaker. In addition, vent may also have acoustic resonances along its length (resonance frequencies $f_r = \frac{c}{2L_e}$, where c is speed of sound and L_e - effective length of the vent taking into account length corrections due to radiation discontinuity)

At low frequencies, complete system (loudspeaker + vented enclosure) is a two degrees of freedom system, thus it has two resonance frequencies.

Attention : Warning

Resonance frequencies of the complete system are different from that of a loudspeaker alone and of a Helmholtz resonator.

- Electrical impedance has two resonances. Frequency for which the value of electrical impedance is minimum (between two resonances) is very close to the resonance frequency of a Helmholtz resonator.
- Pressure response is a high pass filter of order 4 (slope of 24 dB / octave) which is adjusted by comparing the theoretical response with electrical filters of order 4 (alignments). At high frequencies, pressure response may involve bursts due to the internal resonances of the box (which can be damped using absorbent material) and the resonances of the vent.

B. Test your knowledge

1. Exercice : QCM(1)

[*Solution n°5 p 37*]

Loudspeaker mounted in an infinite baffle is assimilated to a rigid oscillating piston. Real part of its radiation impedance is written:

- [ ] $R_{ar} = 2Z_c(ka)$
- [ ] $R_{ar} = 2Z_c(ka)^2$
- [ ] $R_{ar} = Z_c(ka)$
- [ ] $R_{ar} = Z_c(ka)^2$
- [ ] $R_{ar} = Z_c\frac{(ka)}{2}$
- [ ] $R_{ar} = Z_c\frac{(ka)^2}{2}$
- [ ] $R_{ar} = Z_c\frac{(ka)}{4}$
- [ ] $R_{ar} = Z_c\frac{(ka^2)}{4}$

2. Exercice : QCM(2)

[Solution n°6 p 37]

Loudspeaker mounted at the end of a tube is assimilated to a rigid oscillating piston. Section of the tube is identical to that of the piston. Real part of its radiation impedance is written:

- [ ] $R_{ar} = 2Z_c(ka)$
- [ ] $R_{ar} = 2Z_c(ka)^2$
- [ ] $R_{ar} = Z_c(ka)$
- [ ] $R_{ar} = Z_c(ka)^2$
- [ ] $R_{ar} = Z_c\frac{(ka)}{2}$
- [ ] $R_{ar} = Z_c\frac{(ka)^2}{2}$
- [ ] $R_{ar} = Z_c\frac{(ka)}{4}$
- [ ] $R_{ar} = Z_c\frac{(ka^2)}{4}$

3. Exercice : QCM(3)

[Solution n°7 p 38]

Loudspeaker has a resonant frequency fs = 50Hz, Vas = 20l and quality factor Qts = 0.5. This loudspeaker is mounted in a sealed enclosure of volume Vb = 30 l. Resonance frequency and tquality factor of a complete system are:

- ☐ f_s = 64,5 Hz, Q_b = 0,65
- ☐ f_s = 92,4 Hz, Q_b = 0,65
- ☐ f_s = 39,8 Hz, Q_b = 0,65
- ☐ f_s = 64,5 Hz, Q_b = 1,1
- ☐ f_s = 92,4 Hz, Q_b = 1,1
- ☐ f_s = 39,8 Hz, Q_b = 1,1
- ☐ f_s = 64,5 Hz, Q_b = 0,3
- ☐ f_s = 92,4 Hz, Q_b = 0,3
- ☐ f_s = 39,8 Hz,Q_b = 0,3

4. Exercice : QCM(4)

[*Solution n°8 p 38*]

Pressure response of a loudspeaker mounted in a sealed enclosure is:

- ☐ low pass filter with 6 dB / octave slope
- ☐ low pass filter with 12 dB / octave slope
- ☐ low pass filter with 24 dB / octave slope
- ☐ band pass filter with 6 dB / octave slope
- ☐ band pass filter with 12 dB / octave slope
- ☐ band pass filter with 24 dB / octave slope
- ☐ high pass filter with 6 dB / octave slope
- ☐ high pass filter with 12 dB / octave slope
- ☐ high pass filter with 24 dB / octave slope

5. Exercice : QCM(5)

[*Solution n°9 p 38*]

Pressure response of a loudspeaker mounted in a vented enclosure is:

- ☐ low pass filter with 6 dB / octave slope
- ☐ low pass filter with 12 dB / octave slope
- ☐ low pass filter with 24 dB / octave slope
- ☐ band pass filter with 6 dB / octave slope
- ☐ band pass filter with 12 dB / octave slope
- ☐ band pass filter with 24 dB / octave slope
- ☐ high pass filter with 6 dB / octave slope
- ☐ high pass filter with 12 dB / octave slope
- ☐ high pass filter with 24 dB / octave slope

6. Exercice : QCM(6)

[Solution n°10 p 39]

At low frequencies (below the first resonant frequency of a cavity or a vent), a loudspeaker in a vented enclosure is a system :

☐	one degree of freedom
☐	two degree of freedom
☐	three degree of freedom
☐	four degree of freedom

Annex

VII

A. Sign convention for acoustic radiation

1. Bibliography

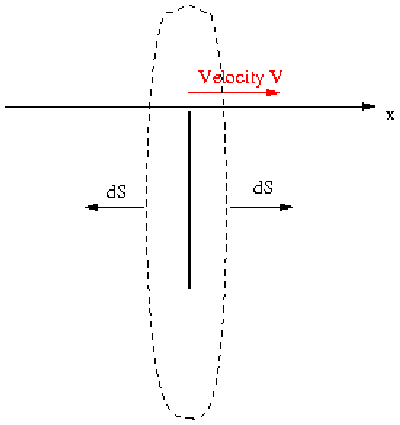

```
1  L L Beranek.
2  Acoustics.
3  American Institute of Physics, 1990.
```

```
1  J Panzer, Akabak,
```

```
1  R.H. Small, Loudspeakers in vented boxes, JAES
```

```
1  L L Beranek and T Mellow, Acoustics : sound fields and transducers,
2  Academic Press, 2012.
```


2. Source radiation



Consider a plane sound source radiating into infinite space. Acoustic volume velocity produced by this source through the surface S is defined by:

$$w = \int \int_S \overrightarrow{v(r)} d\vec{S}$$

where $d\vec{S}$ is equal to $\vec{n}\,dS$. $\vec{n}$ is a normaly oriented vector outside of the volume.

Thus, for positive speed, volume velocity is also positive on the front surface of the source and negative on the rear. Front and rear volume velocities are therefore in phase opposition.

3. Loudspeaker parameters

Loudspeaker parameters are noted in this section

- Electrical parameters of a coil
  - Resistance R_{ec} $[\Omega]$

- Inductance L_{ec} [H]
- Electric impedance of a coil: $Z_{ec} = R_{ec} + j\omega L_{ec}$.

• Mechanical parameters of a mooving part
  - Compliance C_{mm} [m/N]
  - Masse M_{mm} [kg]
  - Resistance R_{mm} [kg=s]
  - Mechanical impedance: $Z_{mm} = R_{mm} + j\omega M_{mm} + \frac{1}{j\omega C_{mm}}$

• Coupling parameters
  - Force factor Bl [T.m] [N/A]
  - Equivalent surface of a membrane S_d [m^2]
  - Equivalent radius of a membrane r_d [m]

4. Loudspeaker parameters

Loudspeaker that is used to illustrate speaker system responses is Visaton 170S whose parameters are:

Rated power 50 W, Maximum power 80 W

Nominal impedance Z 8 Ohm

Frequency response fu-8000 Hz (fu : Lower cut-off frequency depending on a cabinet)

Mean sound pressure level 86 dB (1 W/1 m), Opening angle (-6 dB) 72°/4000 Hz

Excursion limit ± 10 mm

Resonance frequency fs 36 Hz

Magnetic induction 1,0 T, Magnetic flux 314 µWb

Height of front pole-plate 4 mm, Voice coil diameter 25 mm

Height of winding 12,5 mm, Cutout diameter 148 mm

Net weight 1,1 kg

D.C. resistance Rdc 5,9 Ohm

Mechanical Q factor Qms 2,43, Electrical Q factor Qes 0,66, Total Q factor Qts 0,52

Equivalent volume Vas 38 l

E?ective piston area Sd 129 cm^2

Dynamically moved mass Mms 13 g

Force factor Bl 5,4 Tm

Inductance of the voice coil L 1,2 mH

Solution des exercices

> Solution n°1 (exercice p. 4)

- [ ] First order resonant system
- [ ] Two degree of freedom system
- [x] Second order resonant system
- [x] One degree of freedom system

> Solution n°2 (exercice p. 4)

- [ ] Acoustic mass
- [x] Acoustic compliance
- [ ] Second order resonant system
- [ ] Acoustic resistance

> Solution n°3 (exercice p. 5)

- [ ] Acoustic mass
- [ ] Acoustic compliance
- [x] Second order resonant system
- [ ] Acoustic resistance

> Solution n°4 (exercice p. 5)

- [ ] Preserves the mass (does not modify the mass of a piston)
- [x] Increases piston mass
- [ ] Reduces piston mass
- [ ] Increases the apparent area of a piston

> Solution n°5 (exercice p. 30)

- [ ] $R_{ar} = 2Z_c(ka)$
- [ ] $R_{ar} = 2Z_c(ka)^2$
- [ ] $R_{ar} = Z_c(ka)$
- [ ] $R_{ar} = Z_c(ka)^2$
- [ ] $R_{ar} = Z_c\frac{(ka)}{2}$
- [x] $R_{ar} = Z_c\frac{(ka)^2}{2}$
- [ ] $R_{ar} = Z_c\frac{(ka)}{4}$
- [ ] $R_{ar} = Z_c\frac{(ka^2)}{4}$

Zc is the characteristic impedance and is written $Z_c = \rho c$ S where c is the sound speed ρ is the air density S is the piston section.

> Solution n°6 (exercice p. 31)

- [ ] $R_{ar} = 2Z_c(ka)$
- [ ] $R_{ar} = 2Z_c(ka)^2$
- [ ] $R_{ar} = Z_c(ka)$
- [ ] $R_{ar} = Z_c(ka)^2$
- [ ] $R_{ar} = Z_c\frac{(ka)}{2}$
- [ ] $R_{ar} = Z_c\frac{(ka)^2}{2}$
- [ ] $R_{ar} = Z_c\frac{(ka)}{4}$
- [x] $R_{ar} = Z_c\frac{(ka^2)}{4}$

Zc is the characteristic impedance and is written $Z_c = \rho c$ S where c is the sound speed ρ is the air density S is the piston section.

> Solution n°7 (exercice p. 31)

- [x] f_s = 64,5 Hz, Q_b = 0,65
- [ ] f_s = 92,4 Hz, Q_b = 0,65
- [ ] f_s = 39,8 Hz, Q_b = 0,65
- [ ] f_s = 64,5 Hz, Q_b = 1,1
- [ ] f_s = 92,4 Hz, Q_b = 1,1
- [ ] f_s = 39,8 Hz, Q_b = 1,1
- [ ] f_s = 64,5 Hz, Q_b = 0,3
- [ ] f_s = 92,4 Hz, Q_b = 0,3
- [ ] f_s = 39,8 Hz,Q_b = 0,3

> Solution n°8 (exercice p. 32)

- [ ] low pass filter with 6 dB / octave slope
- [ ] low pass filter with 12 dB / octave slope
- [ ] low pass filter with 24 dB / octave slope
- [ ] band pass filter with 6 dB / octave slope
- [ ] band pass filter with 12 dB / octave slope
- [ ] band pass filter with 24 dB / octave slope
- [ ] high pass filter with 6 dB / octave slope
- [x] high pass filter with 12 dB / octave slope
- [ ] high pass filter with 24 dB / octave slope

> Solution n°9 (exercice p. 32)

- [ ] low pass filter with 6 dB / octave slope
- [ ] low pass filter with 12 dB / octave slope
- [ ] low pass filter with 24 dB / octave slope
- [ ] band pass filter with 6 dB / octave slope
- [ ] band pass filter with 12 dB / octave slope
- [x] band pass filter with 24 dB / octave slope
- [ ] high pass filter with 6 dB / octave slope
- [ ] high pass filter with 12 dB / octave slope
- [ ] high pass filter with 24 dB / octave slope

> Solution n°10 (exercice p. 33)

- [ ] one degree of freedom
- [x] two degree of freedom
- [ ] three degree of freedom
- [ ] four degree of freedom