Analysis of electro-mechano-acoustical circuits

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Table des matières

Objectifs

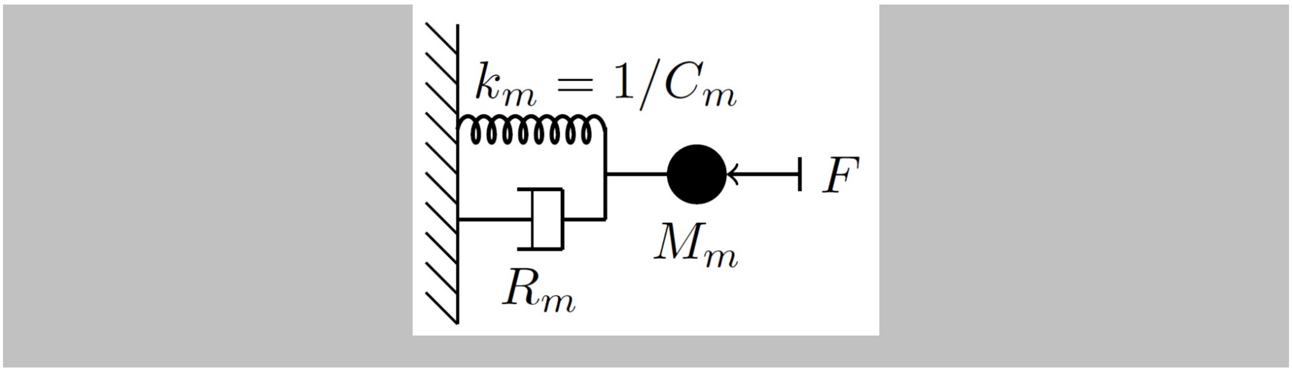
- The objective of this section is to simply model electro-mechano-acoustical systems using tools presented in chapters 2 and 3.
- Three systems will be studied in this section:
  - a first electro-mechanical system: the shaker,
  - a second mechano-acoustical system: the suspended membrane,
  - a third electro-mechano-acoustical system: the electrodynamic loudspeaker.

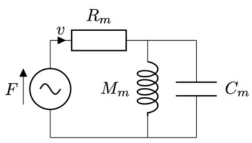
I

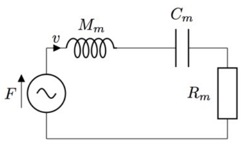
Exercice : Entrance test

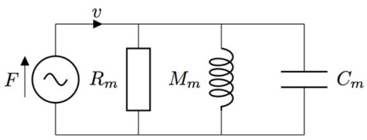
[Solution n°1 p 26]

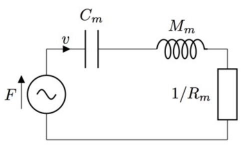
What is the equivalent circuit of the mass-spring-damper system shown below?



- ○ 

- ○ 

- ○ 

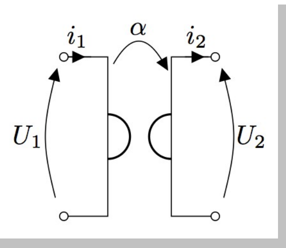
- ○ 

Exercice : Entrance test

II

[Solution n°2 p 26]

Which relations link the electrical quantities at the inputs of the gyrator below?



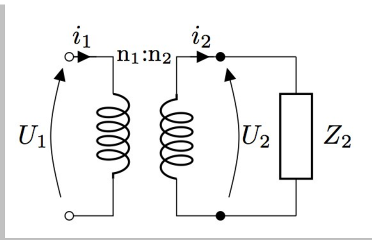
- $\begin{cases} i_1 = \alpha u_1 \\ i_2 = \alpha u_2 \end{cases}$
- $\begin{cases} u_1 = \alpha i_2 \\ i_1 = \alpha u_2 \end{cases}$
- $\begin{cases} u_1 = \alpha i_2 \\ u_2 = \alpha i_1 \end{cases}$
- $\begin{cases} u_2 = \alpha u_1 \\ i_2 = \alpha i_1 \end{cases}$

Exercice : Entrance test

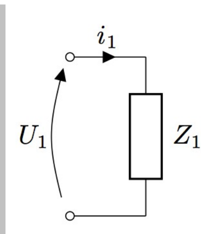
III

[Solution n°3 p 27]

The transformer shown below loaded by the impedance Z_2.



is equivalent to the next one.



Give the expression of Z_1.

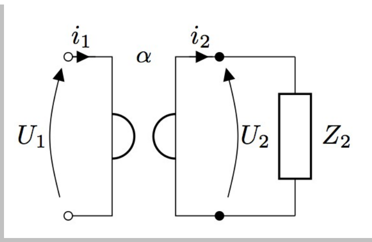
- ○ $Z_1 = \left(\frac{n_1}{n_2}\right)^2 Z_2$
- ○ $Z_1 = \frac{n_1}{n_2} Z_2$
- ○ $Z_1 = \left(\frac{n_1}{n_2}\right)^2 \frac{1}{Z_2}$
- ○ $Z_1 = \frac{n_2}{n_1} Z_2$

Exercice : Entrance test

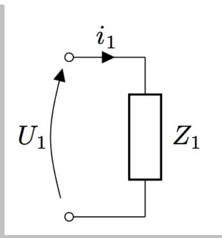
IV

[Solution n°4 p 27]

The gyrator shown below, loaded by the impedance Z_2*,*



is equivalent to the next one.



Give the expression of Z_1

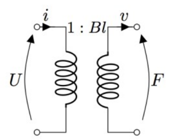
- ○ $Z_1 = \alpha^2 Z_2$
- ○ $Z_1 = \dfrac{\alpha}{Z_2}$
- ○ $Z_1 = \dfrac{Z_2}{\alpha^2}$
- ○ $Z_1 = \dfrac{\alpha^2}{Z_2}$

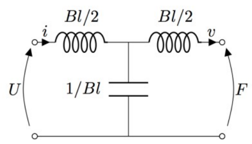
Exercice : Entrance test

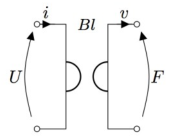
V

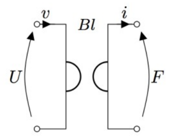
[Solution n°5 p 27]

Which circuit represents the electrodynamic coupling?

- ○ 

- ○ 

- ○ 

- ○ 

VI

Electro-mechanical system: The vibrator

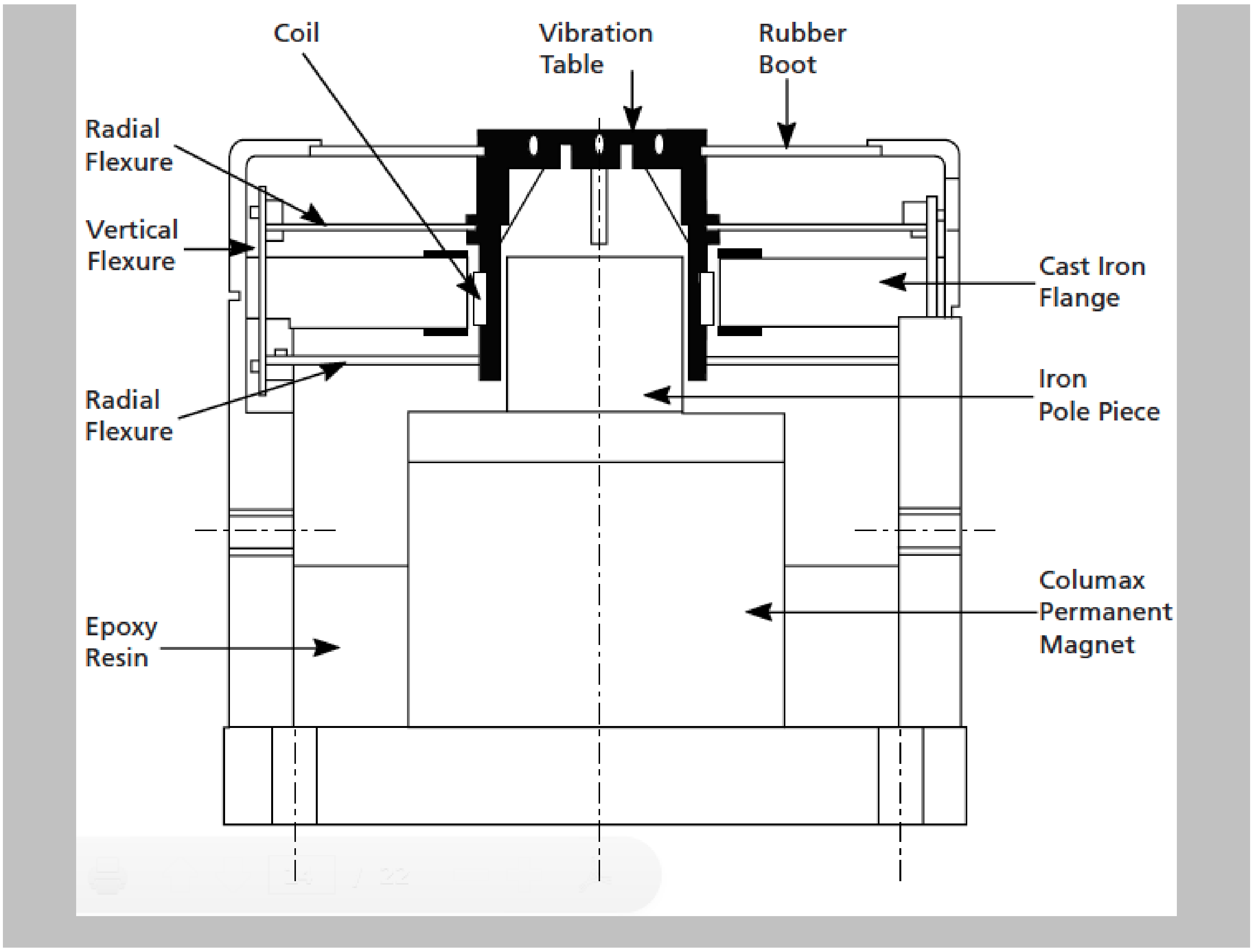
A. General presentation

- The shaker is a system which enables to apply an oscillatory force to an object.
- Due to its construction, it can be used to vibrate relatively heavy structures for vibratory measurements (transparency of dry walls for example).
- It is generally used pour normalised measurements for building acoustics.

B. How it works

- The shaker comprises of an electrodynamic motor, and a mobile mechanical part. The mechanical part is made up of a rigid piston, attached to a frame via a flexible suspension. A piece of rubber on the top of the vibrator creates an air seal.
- The rigid piston is directly coupled (fixed, glued, screwed, etc.) to the target structure.
- The shaker is then excited with an electrical signal from an amplifier.

- When the a current flows through the coil, the induced force F_m (Laplace Force, see chapter 3 section 2) is then applied to a piston which transfers it to the target structure.



C. Modelling

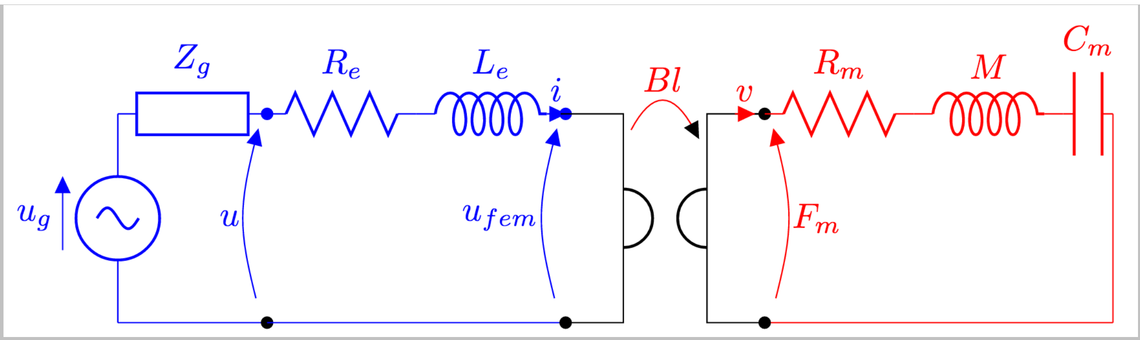
In the following, we suppose that the shaker is not coupled to an exterior structure.

- The motor is composed of an electrical circuit that creates a magnetic field B in which is placed a coil of length ℓ, which has a DC resistance R_e and an electrical inductance L_e .
- The electro-mechanical transduction is characterised by a force factor $B\ell$.
- The mechanical part can be represented by a mass M, corresponding to the mass of the mobile part (rigid piston + coil + suspensions + various parts), a mechanical compliance C_m corresponding to the elastic suspension, and a mechanical resistance R_m which expresses all the mechanical losses of the system (friction in the magnet air gap, losses in the suspension, etc.).
- The electrical system (signal generator + amplifier) can be represented by an equivalent Thevenin generator whose unloaded voltage is U_g and output impedance Z_g .

D. Equations: electro-mechanical circuit

Equivalent electrical circuit

- The equivalent circuit showing the operation of the vibrator is the following:



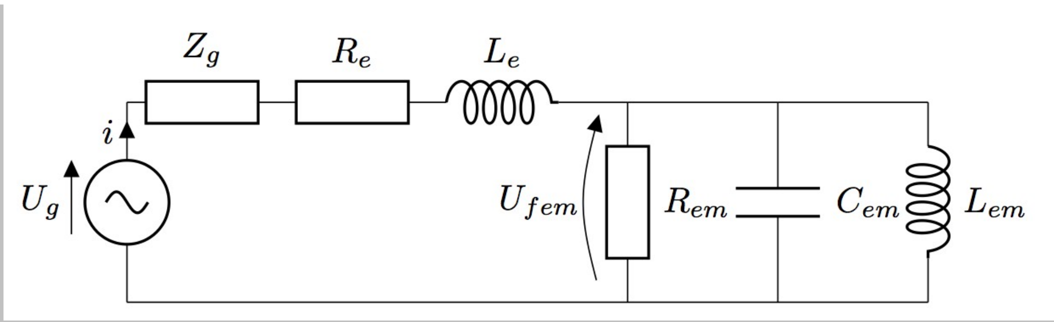
- The equations expressing the electro-mechanical operation of a shaker can be obtained by writing Kirchhoff's circuit laws on either side of the gyrator, as well as the coupling equations of the gyrator:

$$u_g = \left(Z_g + R_e + j\omega L_e\right) i + u_{\text{fem}}$$
$$u_{\text{fem}} = B\ell v$$
$$B\ell i = F_m$$
$$F_m = \left(R_m + j\omega M + \frac{1}{j\omega C_m}\right) v$$

E. Equations: electro-mechanical circuit

Equivalent network, in the electrical domain

- The equivalent network illustrating the operation of the system from an electrical point of view is obtained by bringing all the "mechanical" elements into the "electrical" domain via the gyrator :



- with
  - $R_{em} = \dfrac{B\ell^2}{R_m}$
  - $L_{em} = C_m B\ell^2$
  - $C_{em} = \dfrac{M}{B\ell^2}$

- The previous coupled equations can be reduced to one equation, in the electrical domain:

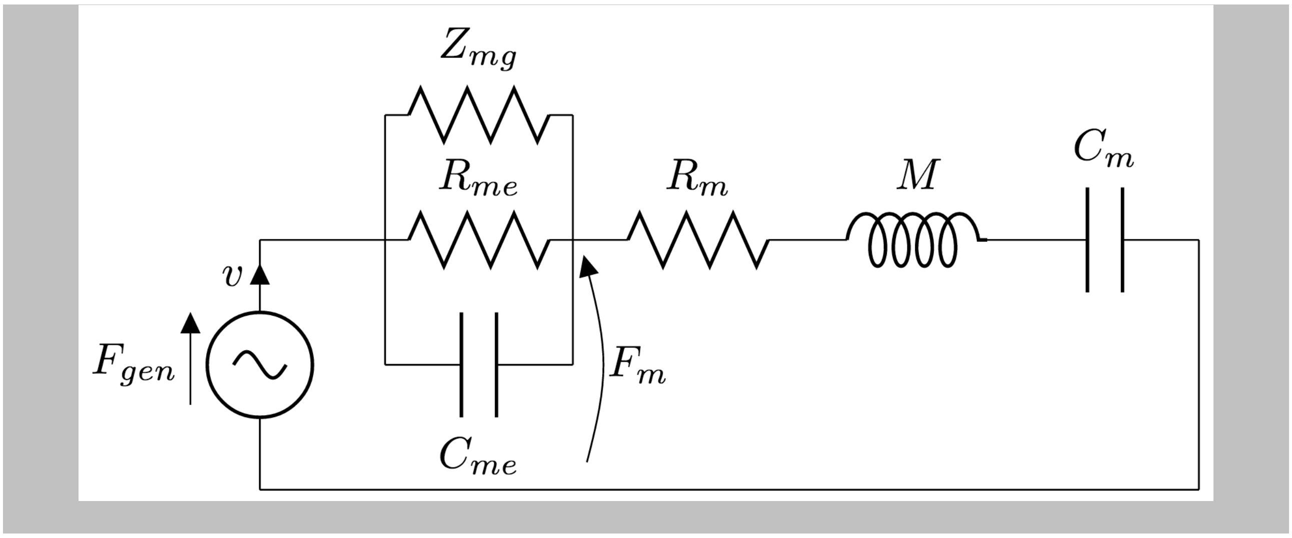
$$u_g = \left(Z_g + R_e + j\omega L_e + \frac{j\omega L_{\text{em}}}{1 + j\omega \frac{L_{\text{em}}}{R_{\text{em}}} - L_{\text{me}} C_{\text{em}} \omega^2} \right) i \qquad (1)$$

-

F. Equations, mechanical schematic

Equivalent network, in the mechanical domain

- The equivalent network describing the operation of the system from a mechanical point of view is obtained by bringing the "electrical" elements into the "mechanical" domain via the gyrator:



- with,
  - $F_{gen} = \frac{B\ell}{Z_g + R_e + j\omega L_e} u_{\text{gen}}$
  - $Z_{mg} = \frac{B\ell^2}{Z_g}$
  - $R_{\text{me}} = \frac{B\ell^2}{R_e}$
  - $C_{\text{me}} = \frac{L_e}{B\ell^2}$
- The previous coupled equations can be reduced to one equation, in the mechanical domain:

$$F_{\text{gen}} = \left(\frac{\frac{R_{\text{me}} Z_{mg}}{R_{\text{me}} + Z_{mg}}}{1 + j \frac{R_{\text{me}} Z_{mg}}{R_{\text{me}} + Z_{mg}} C_{\text{me}} \omega} \right) v$$

G. Transfer function calculation: v_M/u_G

Question

[Solution n°6 p 27]

- By using the previous theoretical elements, give the transfer function's expression $v_m(\omega)/u_g(\omega)$ of the vibrator.
- Draw the module of this transfer function, relative to frequency.
- In the scope of this exercise, we consider that the frequency range is that where the electrical impedance $j\omega L_e$ of the inductance of the coil is negligible relative to the DC resistance R_e of the coil.
- We consider that the source impedance Z_g is negligible relative to the DC resistance, R_e, of the coil.

Mechano-acoustical system: the suspended membrane

VII

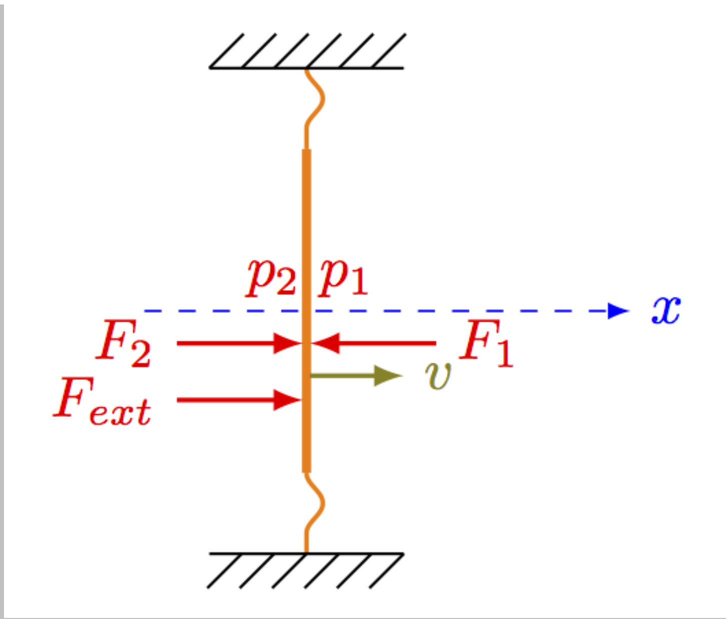
A. Overview

- Here, we consider the behaviour of a vibrating membrane loaded by the air on both sides.
- This configuration corresponds, for example, to a passive radiator in a loudspeaker, or the case of a membrane in an acoustic "bass-trap".

B. Detailed presentation: exciting a suspended membrane via an exterior force

Description of the studied membrane

- Let us consider the case of a circular membrane attached to a rigid structure by a peripheral elastic suspension.
- By applying an exterior mechanical force F_{ext} to the membrane,it will move and may emit sound from its two faces.
- For frequencies situated below its first mode, this membrane can be considered like a mass-spring system comprised of a rigid piston and an elastic peripheral suspension.
- The mechanical elements representing the membrane are thus the total mobile mass M, the compliance $C_m$of the suspensions, and the mechanical resistance R_m mainly due to the viscous losses in the suspension.
- The pressure in front of the membrane (on the right, by convention) and behind (on the left) are noted p_1 and p_2.The associated pressure forces are noted F_1 and F_2. These forces which act upon the membrane correspond to the reaction of the air on the membrane, which moves due to the force F_{ext}.

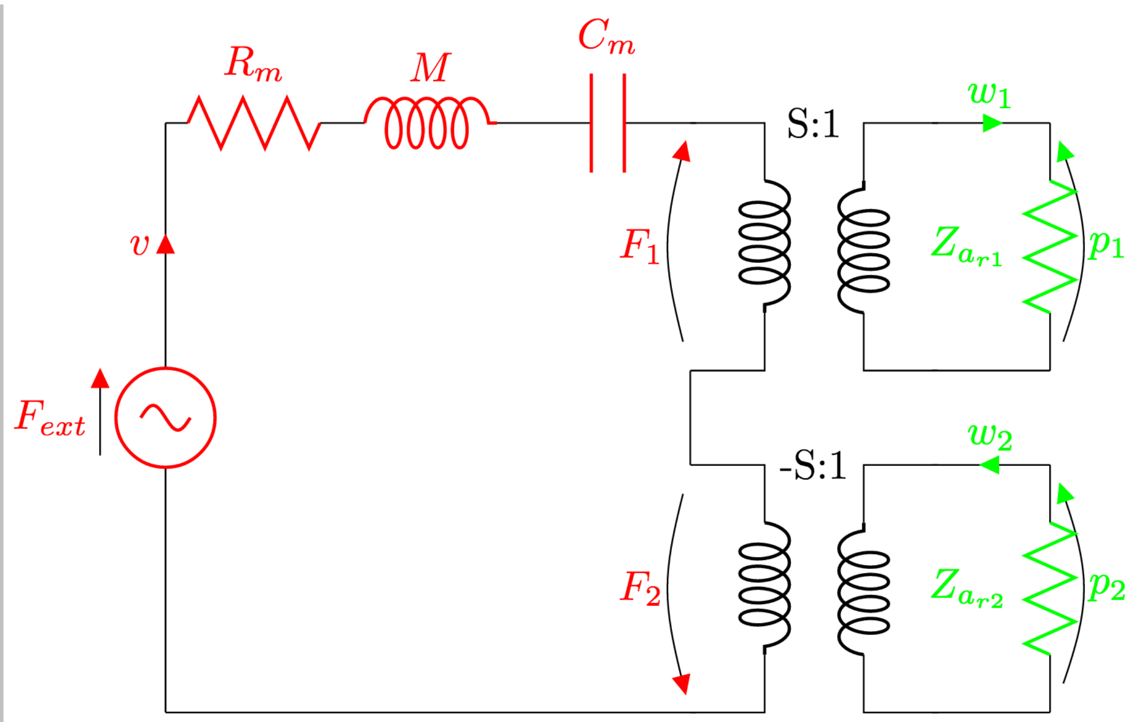


Result of forces on the membrane

C. Equations, mecanico-acoustical network

Mecanico-acoustical network

- Taking the chosen orientations for the force F and velocity v of the membrane into account, the equivalent network of the studied system is the following:



- The impedances Z_{ar1} and Z_{ar2} are the radiation impedances that express the reaction of the eternal medium on the membrane (see section 3.1 "mecanico-acoustical coupling"). *To know more* - p.31

- The behavioural equations of the suspended membrane can be obtained by writing Kirchhoff's voltage law on either side of the transformers:

$$F_m = \left(R_m + j\omega M + \frac{1}{j\omega C_m} + F_1 - F_2\right) v$$

$$F_1 = S\,p_1 \qquad v = \frac{w_1}{S}$$

$$F_2 = S\,p_2 \qquad v = -\frac{w_2}{S}$$

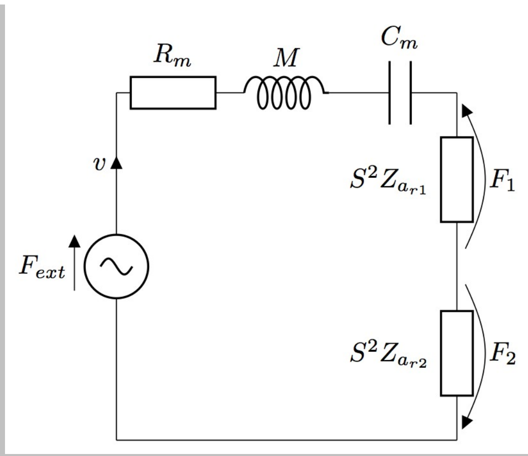
$$p_1 = Z_{ar1} w_1$$

$$p_2 = Z_{ar2} w_2$$

D. Equations, mechanical network

Equivalent mechanical network

- The equivalent network describing the operation of the system from a mechanical point of view is obtained by bringing the "acoustical" elements into the "mechanical" domain via the transformers.



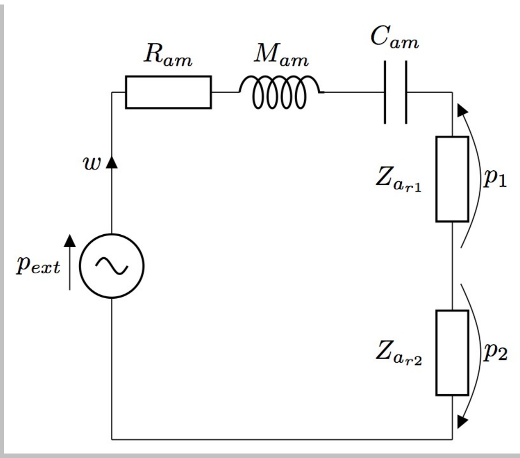
- The previous coupling equations can be reduced to one equation, in the mechanical domain:

$$F_{ext} = \left(j\omega M + \frac{1}{j\omega C_m} + R_m + S^2 Z_{ar1} + S^2 Z_{ar2}\right) v$$
.

E. Equations - acoustical network

Equivalent network, in the acoustical domain

- The equivalent network expressing the operation of the system from an acoustical point of view is obtained by bringing the "mechanical" elements into the "acoustical" domain via the transformers.



- with
  - $p_{ext} = F/S$
  - $M_{am} = M/S^2$
  - $R_{am} = R_m/S^2$
  - $C_{am} = C_m S^2$
- The previous coupling equations can be reduced to one equation, in the acoustical domaine:

$$p_{ext} = \left(j\omega M_{am} + \frac{1}{j\omega C_{am}} + R_{am} + Z_{ar1} + Z_{ar2}\right) w$$

.

F. Calculation of the mechanical admittance

Question

[Solution n°7 p 28]

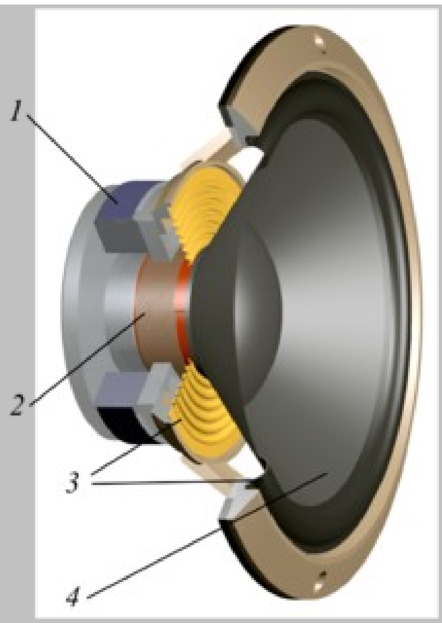
- By considering that the radiation impedance loading the two faces of the membrane suspended by its peripheral suspension can be reduced to two radiation masses, give the expression of the mechanical admittance $Y_m = v/F$ presented by this membrane.
- Plot the module of this transfer function relative to the frequency.

Electro-mechano-acoustical system:

VIII

A. Overview

Overview

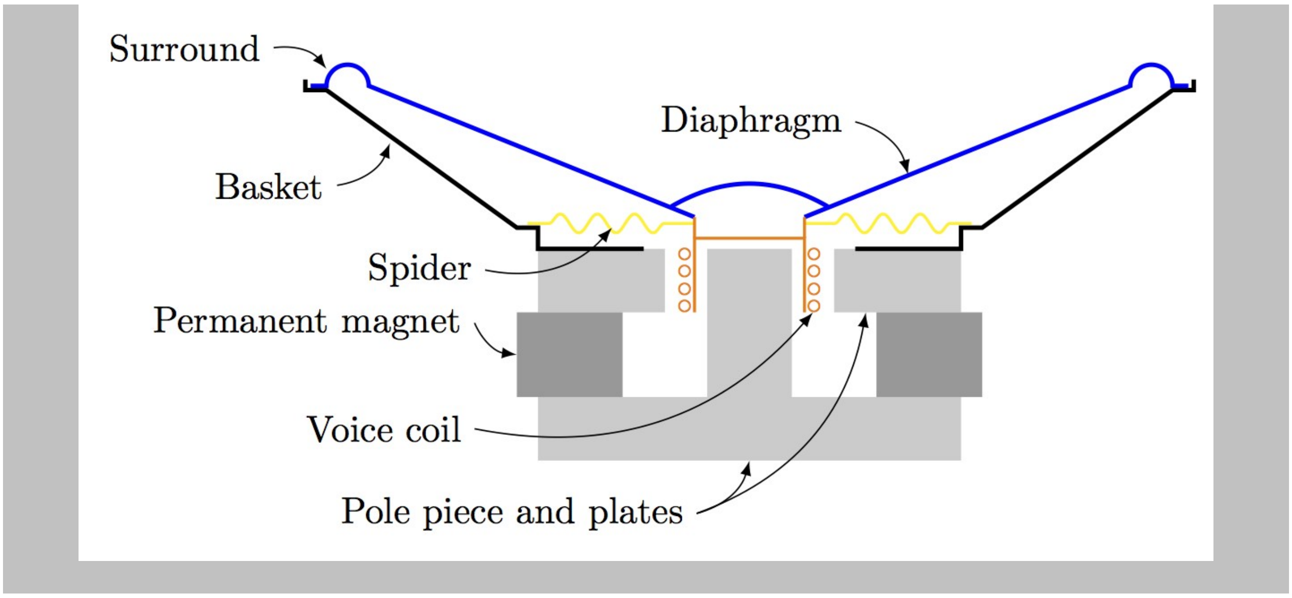


- We consider here, the case of an electrodynamic loudspeaker with a voice coil.
- The electrodynamic loudspeaker is an electro-mechano-acoustic system commonly composed of
  - a solid chassis, usually called the **basket**,
  - a **membrane suspended** in the basket via a peripheral elastic surround,

- an **electrodynamic motor**, which is composed of a magnetic circuit and a moving coil.

B. Detailed description

Detailed description



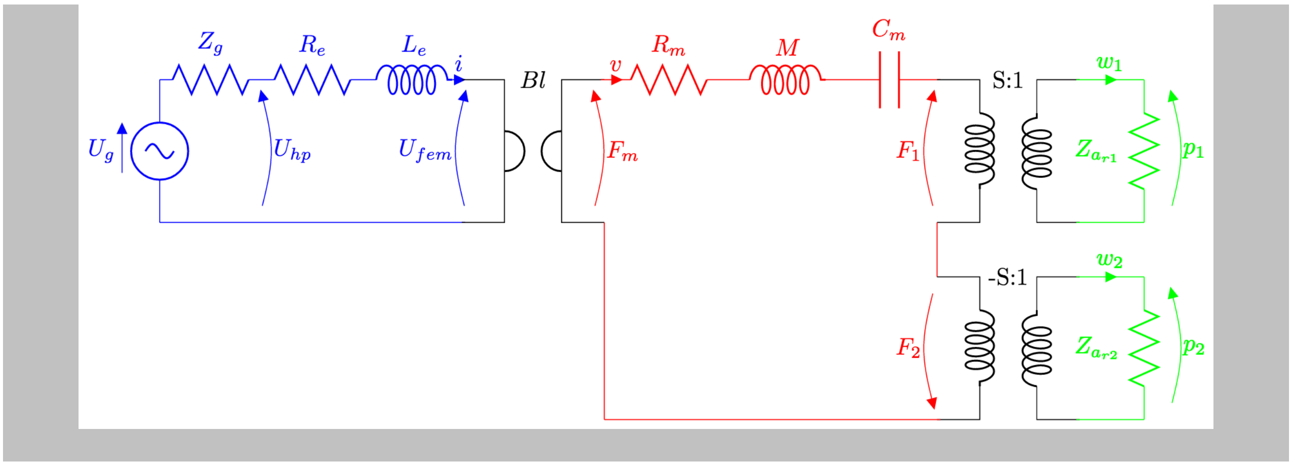
- The loudspeaker motor is composed of a magnetic circuit which creates a magnetic field B in which is placed a coil of length ell, of resistance R_e and electrical inductance L_e.
- The electro-mechanical transduction is characterized by the force factor $B\ell$.
- The mechanical part can be represented by associating an mass M, corresponding to the mass of the mobile part (membrane + coil + suspension), of a mechanical compliance $C_m$corresponding to the elastic suspension, and a mechanical resistance R_m which expresses the totality of the mechanical losses of the system (friction in the air gap, losses in the suspensions, etc.).
- The loudspeaker radiates into an infinite space thanks to its two faces. Each of these two face, of surface S

  is thus loaded by an acoustic radiation impedance Z_{ar}.

C. Equations, mechano-electro-acoustical network

Mechano-electro-acoustical network

- The equivalent network illustrating the operation of the electrodynamic loudspeaker is the following:



- On this network, u_g and Z_g represent the unloaded voltage and output impedance of the sources equivalent Thevenin generator.
- The equations describing the electro-mechano-acoustical behaviour of the loudspeaker can be obtained by applying Kirchhoff's voltage law on either side of the gyrator and transformers, as well as the coupling equations for the gyrator and transformers:

$$u_{\text{gen}} = \left(Z_{\text{gen}} + R_e + j\omega L_e\right) i + u_{\text{fem}}$$

$$u_{\text{fem}} = B\ell v$$

$$B\ell i = F_m$$

$$F_m = \left(R_m + j\omega M + \frac{1}{j\omega C_m} + F_1 - F_2\right) v$$

$$F_1 = S\, p_1 \qquad v = \frac{w_1}{S}$$

$$F_2 = S\, p_2 \qquad v = -\frac{w_2}{S}$$

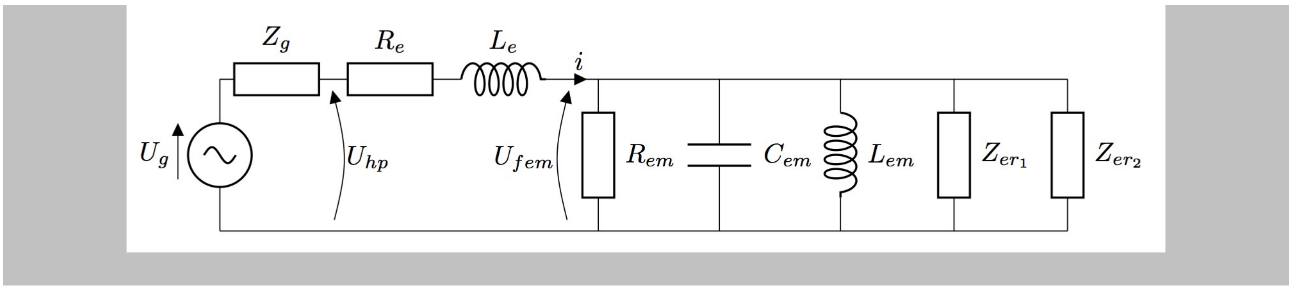
$$p_1 = Z_{ar1} w_1$$

$$p_2 = Z_{ar2} w_2$$

D. Equations, equivalent electrical network

Equivalent network

- The equivalent network form an electrical point of view can be obtained by passing the "mechanical" and "acoustic" elements into the electrical domain through the gyrator:



- with

- $R_{\text{em}} = \dfrac{B\ell^2}{R_m}$
- $L_{\text{em}} = C_m B\ell^2$
- $C_{\text{em}} = \dfrac{M}{B\ell^2}$
- $Z_{er1} = \dfrac{B\ell^2}{S^2 Z_{ar1}}$
- $Z_{er2} = \dfrac{B\ell^2}{S^2 Z_{ar2}}$

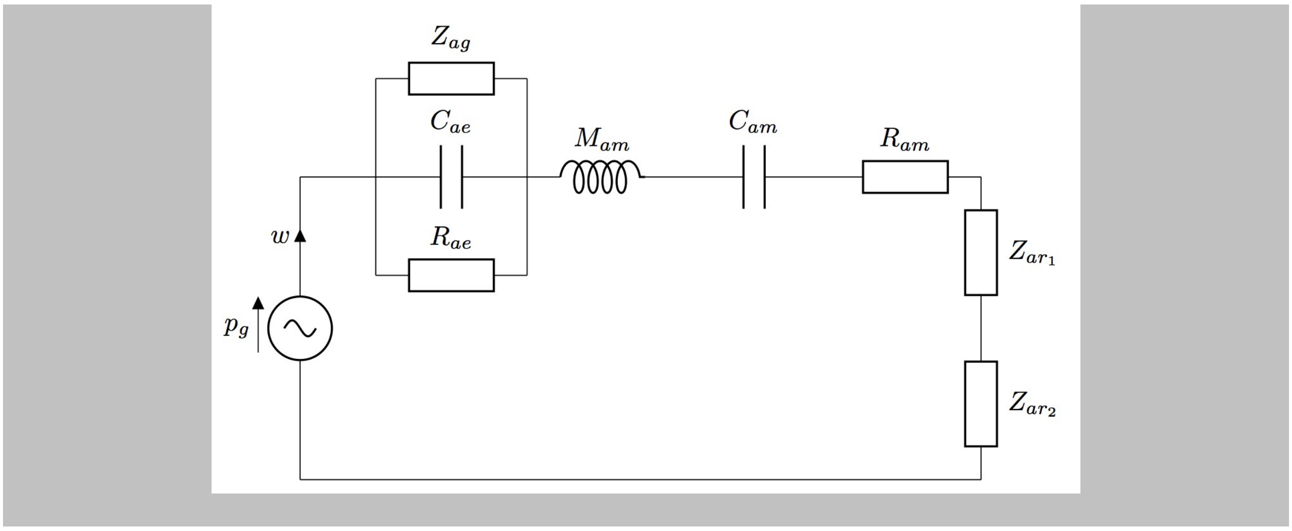
- The previous coupling equations can be reduced to one equations, in the electrical domain:

$$u_g = \left(Z_g + R_e + j\omega L_e + \frac{1}{\dfrac{1}{R_{\text{em}}} + j\omega C_{\text{em}} + \dfrac{1}{j\omega L_{\text{em}}} + \dfrac{1}{Z_{er1}} + \dfrac{1}{Z_{er2}}} \right) i$$

E. Equations, equivalent acoustic network

Equivalent acoustical network

- The equivalent network form an acoustical point of view can be obtained by passing the "mechanical" and "electrical" elements into the acoustical domain through the transformers:



- with
- $p_g = \dfrac{B\ell}{S\left(Z_g + R_e + j\omega L_e\right)} u_g$
- $Z_{ag} = \dfrac{B\ell^2}{S^2 Z_g}$
- $C_{ae} = \dfrac{L_e S^2}{B\ell^2}$
- $R_{ae} = \dfrac{B\ell^2}{S^2 R_e}$

- $M_{am} = \dfrac{M}{S^2}$
- $R_{am} = \dfrac{R_m}{S^2}$
- $C_{am} = C_m S^2$.

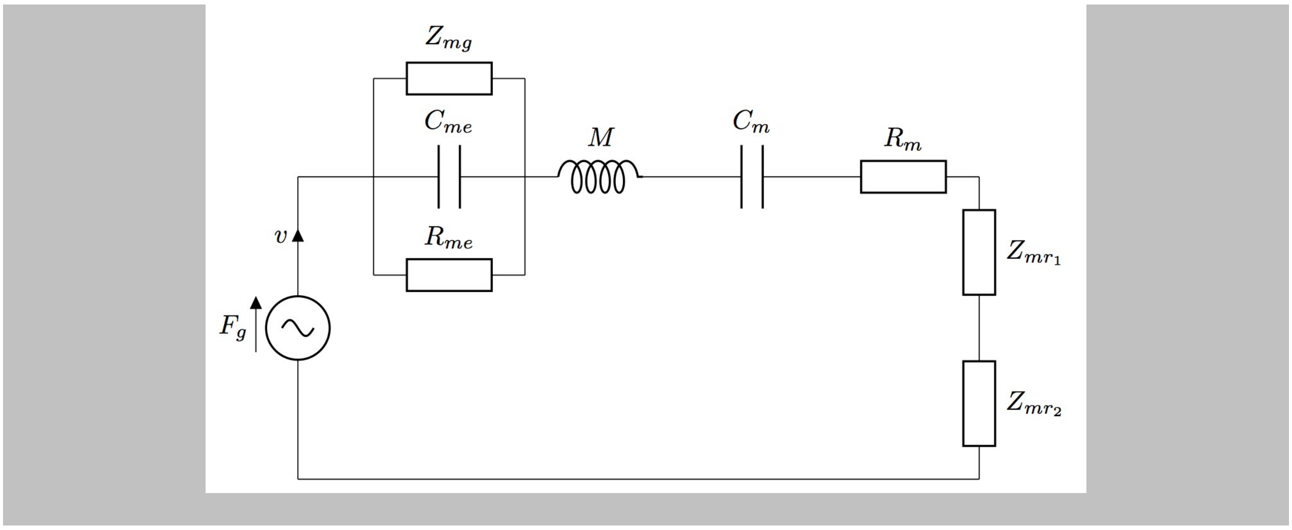
- The previous coupling equations can be reduced to one equations, in the acoustical domain:

$$p_g = \left[\left(\frac{1}{\frac{1}{Z_{ag}} + j\omega C_{ae} + \frac{1}{R_{ae}}}\right) + j\omega M_{am} + \frac{1}{j\omega C_{am}} + R_{am} + Z_{ar1} + Z_{ar2}\right] w$$

F. Equations, equivalent mechanical network

Equivalent mechanical network

- The equivalent circuit from a mechanical point of view can be obtained by passing the "electrical" elements through the gyrator, and the "acoustical" elements into the mechanical domain through the transformers:



- with
  - $F_g = \dfrac{B\ell}{Z_g + R_e + j\omega L_e} u_g$
  - $Z_{mg} = \dfrac{B\ell^2}{Z_g}$
  - $C_{\mathrm{me}} = \dfrac{L_e}{B\ell^2}$
  - $R_{\mathrm{me}} = \dfrac{B\ell^2}{R_e}$
  - $Z_{mr1} = Z_{ar1} S^2$
  - $Z_{mr1} = Z_{ar1} S^2$

- The previous coupling equations can be reduced to one equations, in the mechanical domain:

$$F_g = \left[\left(\frac{1}{\frac{1}{Z_{mg}} + j\omega C_{\mathrm{me}} + \frac{1}{R_{\mathrm{me}}}} \right) + j\omega M + \frac{1}{j\omega C_m} + R_m + Z_{mr1} + Z_{mr2} \right] v .$$

G. Calculation of the electrical input impedance of an electrodynamic loudspeaker

Question

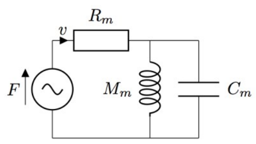
[Solution n°8 p 29]

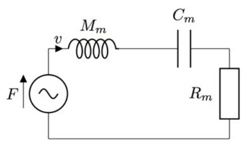
By considering that the radiation impedances loading the two faces of the membrane can be reduced to two simple masses M_{ar}, give the expression of the electrical input impedance of the loudspeaker $Z_{lsp} = u_{lsp}/i$.

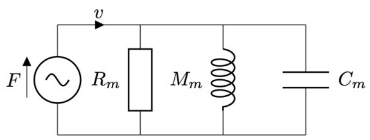
Plot the module of this transfer function relative to the frequency.

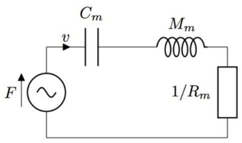
Solution des exercices

> Solution n°1 (exercice p. 6)

○ 

◉ 

○ 

○ 

> Solution n°2 (exercice p. 7)

○ $\begin{cases} i_1 = \alpha u_1 \\ i_2 = \alpha u_2 \end{cases}$

○ $\begin{cases} u_1 = \alpha i_2 \\ i_1 = \alpha u_2 \end{cases}$

◉ $\begin{cases} u_1 = \alpha i_2 \\ u_2 = \alpha i_1 \end{cases}$

○ $\begin{cases} u_2 = \alpha u_1 \\ i_2 = \alpha i_1 \end{cases}$

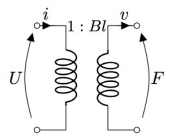
> Solution n°3 (exercice p. 8)

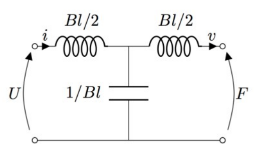
- ◉ $Z_1 = \left(\frac{n_1}{n_2}\right)^2 Z_2$
- ○ $Z_1 = \frac{n_1}{n_2} Z_2$
- ○ $Z_1 = \left(\frac{n_1}{n_2}\right)^2 \frac{1}{Z_2}$
- ○ $Z_1 = \frac{n_2}{n_1} Z_2$

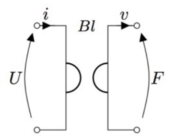
> Solution n°4 (exercice p. 9)

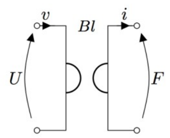
- ○ $Z_1 = \alpha^2 Z_2$
- ○ $Z_1 = \frac{\alpha}{Z_2}$
- ○ $Z_1 = \frac{Z_2}{\alpha^2}$
- ◉ $Z_1 = \frac{\alpha^2}{Z_2}$

> Solution n°5 (exercice p. 10)

- ○ 

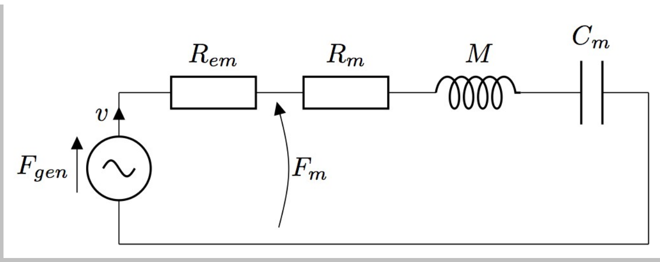
- ○ 

- ◉ 

- ○ 

> Solution n°6 (exercice p. 15)

After taking into account the hypothesis, the equivalent circuit of the vibrator in the mechanical domain becomes



With $F_g = \frac{B\ell}{R_e} u_g$

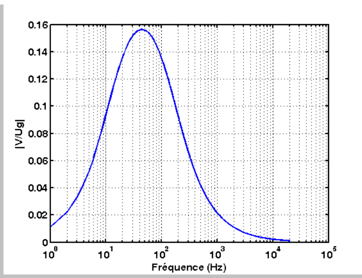
By applying Kirchhoff's voltage law to the circuit, the following expression can be obtained:

$$\frac{v_m}{F_g} = \frac{jC_m\omega}{1 + j\left(R_m + R_{\text{em}}\right)C_m\omega - MC_m\omega^2},$$

or

$$\frac{v_m}{u_g} = \frac{R_e}{B\ell}\frac{jC_m\omega}{1 + j\left(R_m + R_{\text{em}}\right)C_m\omega - MC_m\omega^2}.$$

The representation of this frequency response is therefore the following:



The vibrator has a resonant behavior: for a given input voltage, the velocity of the mobile part is maximal at resonance. $f_r = \frac{1}{2\pi}\frac{1}{\sqrt{MC_m}}$.

> Solution n°7 *(exercice p. 19)*

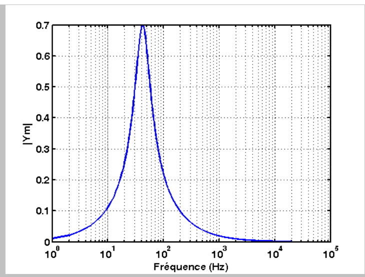
The electrical admittance $Y_m = v/F$ is written:

$$Y_m = v/F = \frac{1}{j\omega\left(M + M_{ar1}S^2 + M_{ar2}S^2\right) + \frac{1}{j\omega C_m} + R_m}$$

Or:

$$Y_m = v/F = \frac{j\omega C_m}{1 + j\omega C_m R_m - \omega^2\left(M + M_{ar1}S^2 + M_{ar2}S^2\right)C_m}$$

A representation of the module of the admittance relative to frequency is the following:



> Solution n°8 *(exercice p. 25)*

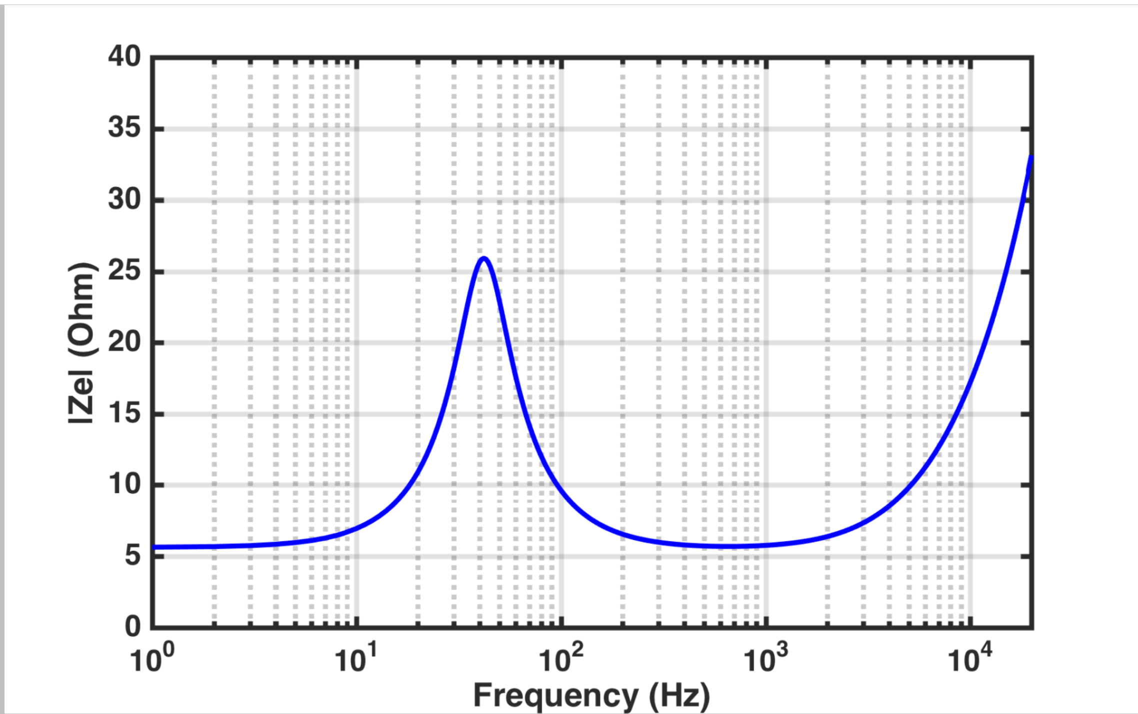
The electrical impedance of the loudspeaker is written:

$$Z_{hp} = R_e + j\omega L_e + \frac{j\omega L_{\text{em}}}{1 + j\omega \frac{L_{\text{em}}}{R_{\text{em}}} - \left(C_{\text{em}} + C_{er1} + C_{er2}\right) L_{\text{em}}\omega^2}$$

with

$$C_e r = \frac{B\ell^2}{S^2 M_{ar}}$$

A representation of the module of the admittance relative to frequency is the following:



This graph allows the identificatin of certain electro-mechanical parameters. In particular:

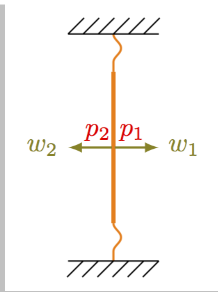
- The module of the impedance tends towards the DC resistance R_e in low frequency.
- The slope of the curve as frequency increases corresponds to the value of the inductance L_e.
- The frequency of the resonance peak corresponds to the resonance of the mechanical part.

$$f_r = \frac{1}{2\pi} \frac{1}{\sqrt{L_{\text{em}} \left(C_{\text{em}} + C_{er1} + C_{er2}\right)}} = \frac{1}{2\pi} \frac{1}{\sqrt{C_m \left(M + M_{ar1} S^2 + M_{ar2S^2}\right)}}$$

Contenus annexes

- To know more: Radiation impedance

By definition, the acoustical radiation impedance of a vibrating piston is the ratio between the pressure p applied to the surface S of the piston and the volume velocity w normal to this surface.

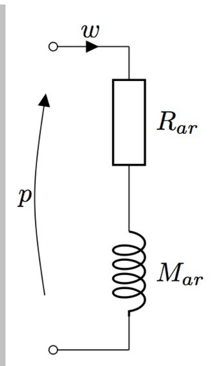


- The front and rear radiation impedances loading the faces of the membrane are thus written:

$$Z_{ar1} = \frac{p_1}{w_1} \quad (1)$$

$$Z_{ar2} = \frac{p_2}{w_2} \quad (2)$$

In low frequency, the equivalent electrical representation of the radiation impedance is the following



where M_{ar} represents the radiation mass, and where R_{ar} represents the radiation resistance (loss).