Electrostatic transduction

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Table des matières

Electrostatic transduction

I

A. Test

Exercice 1 : Question 1

[Solution n°1 p 17]

Ohms law states that:

- ○ The electrical potential across a resistor is directly proportional to the current flowing through it.
- ○ The resistance of a resistor increases if the voltage drop across it increases.
- ○ The resistance of a resistor is proportional to the product of the voltage across the resistor and the current flowing through it.
- ○ The voltage across a resistor is independant of the current flowing through it.

Exercice 2 : Question 2

[Solution n°2 p 17]

The equivalent resistance R_{eq} of two resistors in parallel is written:

- ○ $R_1 + R_2$
- ○ R_1R_2
- ○ $R_1R_2/(R_1+R_2)$
- ○ $R1/(R_2-R_1)$

Exercice 3 : Question 3

[Solution n°3 p 17]

In the case of a sinusoidal voltage $u(t) = U_0 \cos(\omega_0 t + \phi)$, with U_0 the voltage peak value, t the time, ω_0 the angular frequency and ϕ the phase. What is the RMS voltage of this signal ?

- ○ U_0/ϕ
- ○ $U_0/2$
- ○ $U_0/\sqrt{2}$
- ○ $U_0\cos(\omega_0 t)$

Exercice 4 : Question 4

[Solution n°4 p 18]

In the harmonic domain, how is the impedance of an inductor, of inductance L, expressed?

- ◯ L
- ◯ $jL\omega$
- ◯ $1/jL\omega$
- ◯ $L\omega$

Exercice 5 : Question 5

[Solution n°5 p 18]

A circuit composed of a resistor, a capacitor and an inductor in series is:

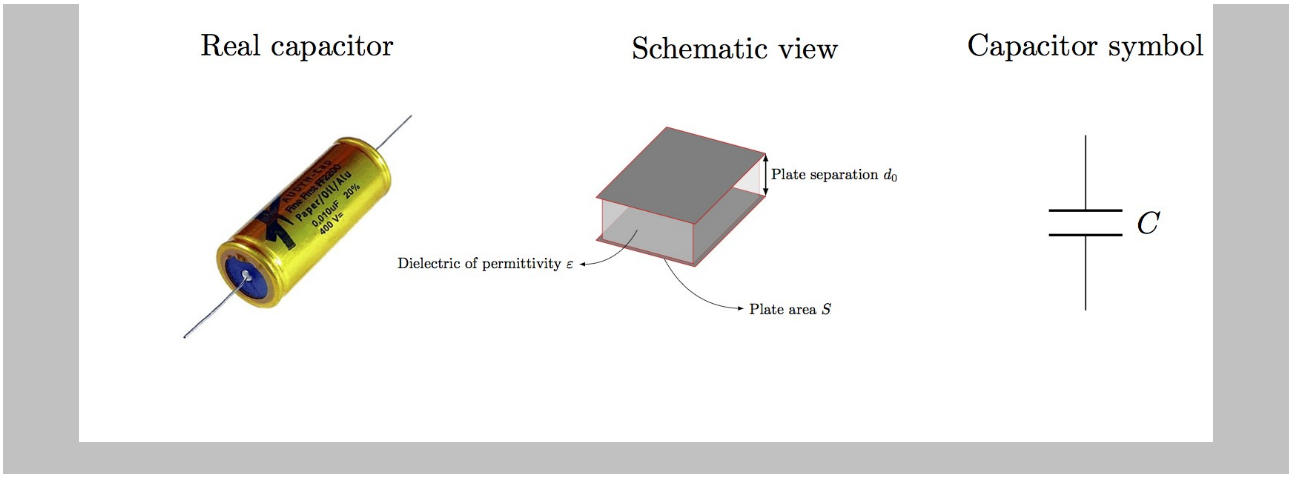
- ◯ an active circuit
- ◯ a loss less circuit
- ◯ a circuit with a period of 24 hours
- ◯ a resonant circuit

B. Generalities and reminders

1. The capacitor

a) Structure of a capacitor

A capacitor is composed of two electrodes with a surface S, separated by an insulator (or dielectric) of thickness d_0.



b) Charging circuit of a capacitor

The capacitor is connected to a source generating a voltage U_0. The resistor R_c in the circuit represents the total resistance of the circuit (source resistance, connector resistance, and also the load resistance).

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c) Charge of a capacitor

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d) Capacity of a capacitor

The relation between the amount of charge Q on the positive electrode, and the voltage drop U_0 across the two electrodes of a capacitor is the electrical capacity C_0 of the capacitor:

$$C_0 = \frac{Q}{U_0}$$

The capacity is always a positive quantity and is measured in Farads.

The higher the capacity, the higher the charge that can be stocked on the positive electrode for a given polarisation voltage U_0.

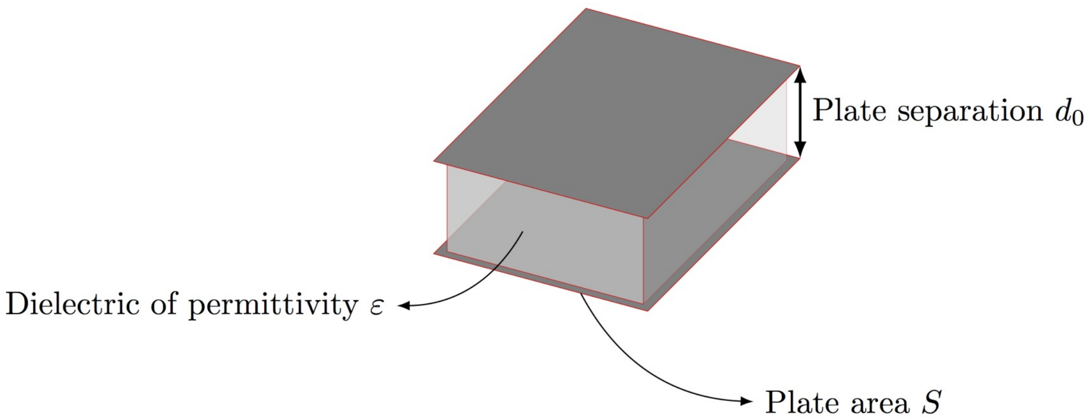
Remarque : Remark

The higher the capacity, the lower the voltage drop for a given quantity of charges.

e) Capacity

The capacity of a circuit depends on the geometrical characteristics of the capacitor (surface area of, and distance between the electrodes) as well as the dielectric properties (permittivity) ε, following the law:

$$C_0 = \frac{\varepsilon S}{d_0}$$



Remarque : Remark

We can see that the capacity of a capacitor varies with the inter electrode distance d0. This phenomenon is used in electrostatic transduction.

f) Electrostatic force

The two electrodes of a charged capacitor carry equal amounts of opposed charges. They are therefore exposed to an electrostatic force F_e and are attracted together.

The electrostatic force F_e acting across the two electrodes is written:

$$F_e = \frac{1}{2}\frac{q}{C(}$$

Complément : To go further

http://res-nlp.univ-lemans.fr/NLP_S_M07_G01_01/co/NLP_S_M07_G01_01_web.html[1]

2. General principle of electrostatic transduction

a) Description of an electrostatic transducer

The electrostatic transduction is usually done with a **variable capacitor**.

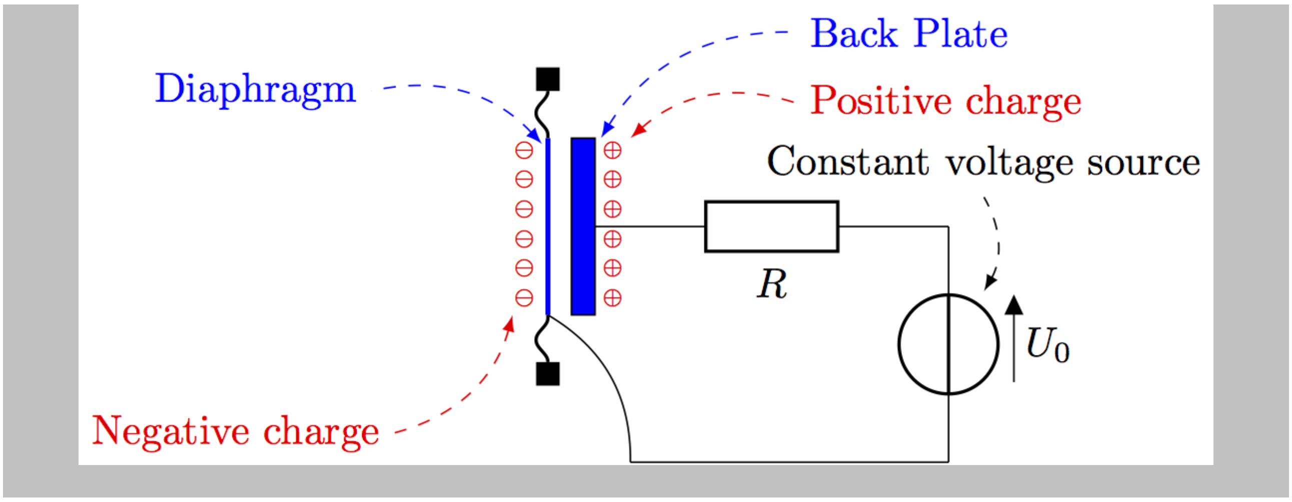
In its most simple form, this capacitor is made up of two plane electrodes. One of these electrodes is mobile and can move towards or away from the fixed electrode.

Exemple : Example

This photo represents two electrostatic microphone capsules. Here, the membrane acts as the mobile electrode of the transducer (visible on the left capsule). The perforated rear electrode is visible on the right capsule due to the removal of the membrane. This electrode is the transducers immobile electrode

b) Polarisation of an electrostatic transducer

Once this capacitor is inserted in an electrical circuit, the two electrodes carry opposed electrical charges. The result, on one hand is a voltage drop across the electrodes, and on the other, the existence of an electrostatic force that attracts them towards each other.



c) Principle of an electrostatic sensor

recomposition.swf

1 - http://res-nlp.univ-lemans.fr/NLP_S_M07_G01_01/co/NLP_S_M07_G01_01_web.html

d) Principle of an electrostatic generator

Animation2.swf

C. Electrostatic coupling equations

1. Objective

This part is dedicated to the coupling equation of electrostatic transduction, in the case of a **generator** (case where an electrical signal moves the mobile electrode of the transducer) and the case of a **receiver** (case where the movement of the electrode generates an electrical signal).

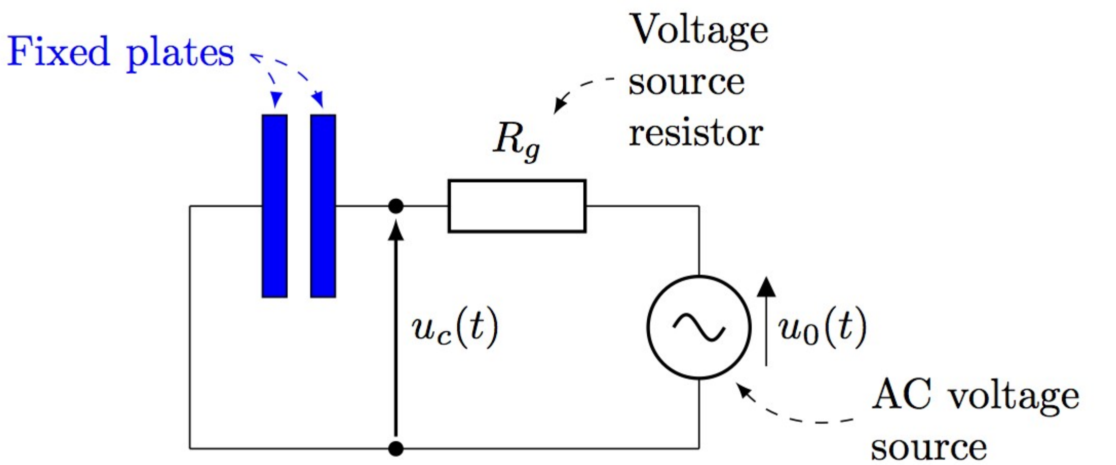
2. Generator: electro-mechanical coupling

a) Electrostatic force

i Electrostatic force applied between two electrodes

To introduce the electro mechanical coupling equation, the two electrodes of the electrostatic transducer are considered immobile. The transducer is thus a simple capacitor. This capacitor is connected to a variable voltage source, with an output impedance R_g and no load voltage $u_0(t)$.

The variable voltage from the source creates a variation of the electrical state of the system. This is seen via a change in the quantity of charge stocked on the electrodes. The consequence of this, is a modification of the electrostatic force applied between the two electrodes. Thus the two electrodes are attracted to each other.



ii Expression of the electrostatic force

The quantity of charges stocked on the electrode is given by $q(t) = C_0 u_c(t)$.

The resulting electrostatic force can be written:

$$F_e(t) = \frac{1}{2}\frac{q^2}{C_0}$$

This formula expresses the **electro-mechanical coupling**, as an AC electrical signal generates a force that may move the mobile electrode of the transducer.

This transduction is intrinsically non linear as the resulting force is proportional to the square of the excitation voltage: so if q(t) is positive or negative, the electrostatic force results in the electrodes moving together.

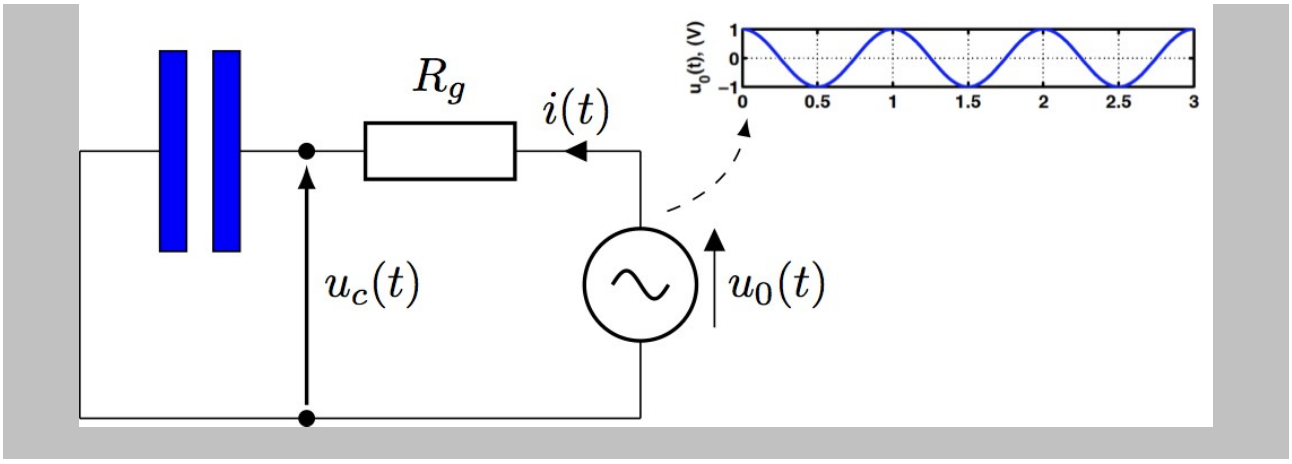
b) Non linear behaviour of the electrostatic coupling

i Excitation voltage

Consider the input voltage $u_0(t)$ of frequency f_0 delivered by an unloaded generator,

$$u_0(t) = U_M \cos$$

where U_M is the RMS value of this voltage and where $\omega_0 = 2\pi f_0$ is the angular frequency corresponding to f_0.

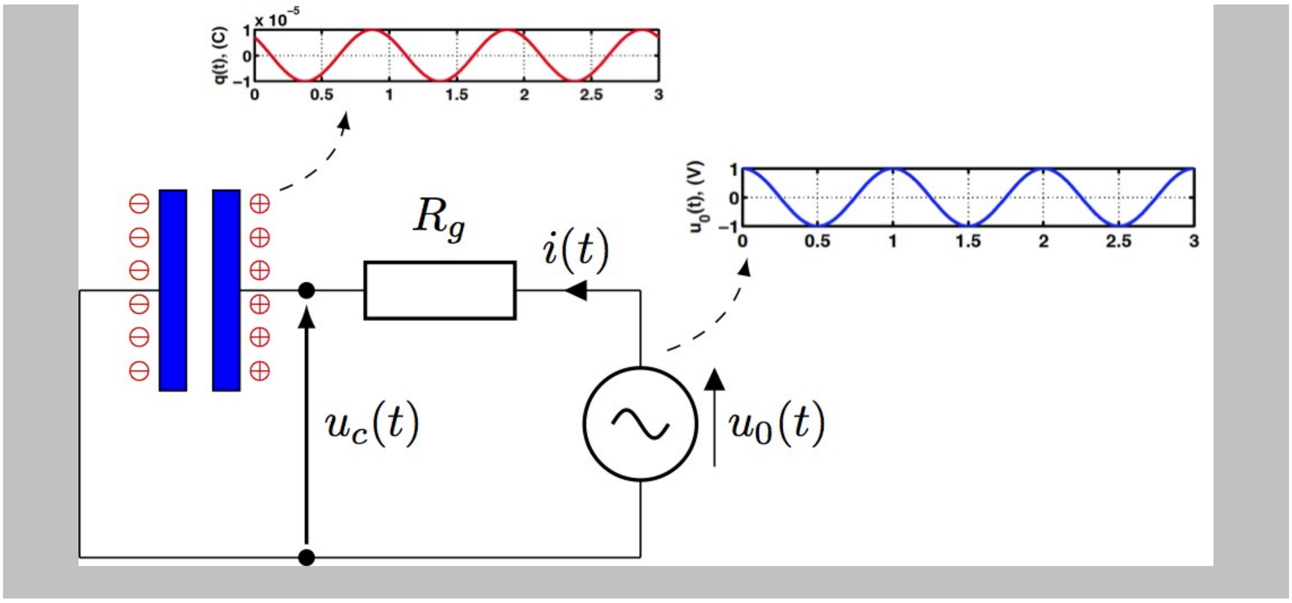


ii Electrical charge

Once a steady state is attained, the charges q(t) are subject to a sinusoidal variation at a frequency f_0 following the law:

$$q(t) = C_0 U_M \cos$$

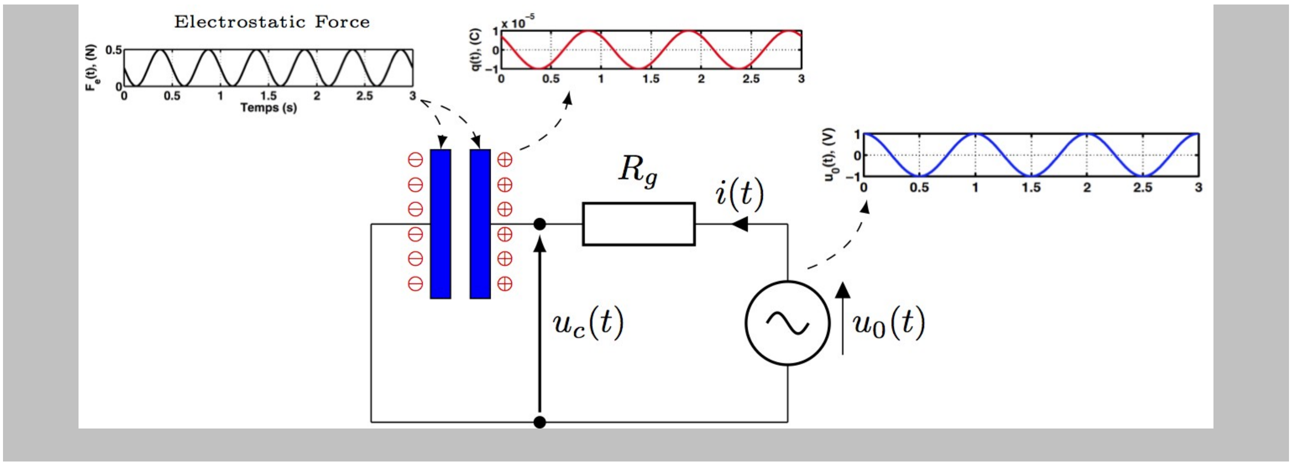
where $\varphi_0 = Arctg(R_g C_0 \omega_0)$ represents the phase difference between the charge q(t) and the input voltage $u_0(t)$.



iii Electrostatic force

The electrostatic force F_e acting on the two electrodes is therefore written

$$F_e(t) = \frac{1}{2}\frac{q^2(t)}{C_0 d_0} = \frac{C_0 U_M^2}{2d_0}\cos^2(\omega t + \varphi) = \underbrace{\frac{C_0 U_M^2}{4d_0}}_{\text{Constant force}} + \underbrace{\frac{C_0 U_M^2}{4d_0}\cos 2(\omega t + \varphi)]}_{\text{Harmonic force oscillating at } 2f}$$

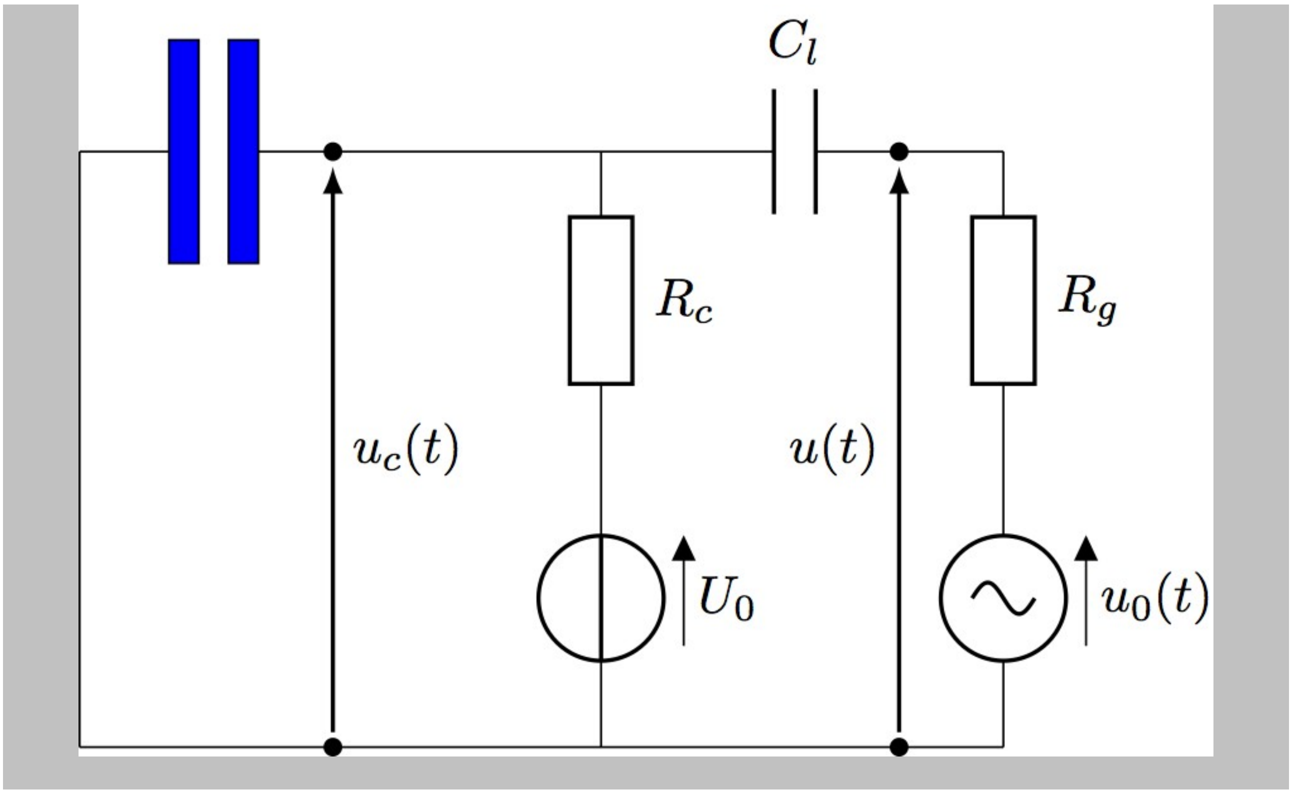


In the case of an electrostatic generator, the electrostatic force creates the movement of the mobile electrode, which generates the acoustical signal. Therefore, a variation of the electrical charge at a frequency f_0 causes an acoustical signal of 2 f_0: it is a non linear signal.

c) Linearization of th electrostatic coupling

i Linearization by polarisation

A common way of linearising the coupling is to use a DC polarisation voltage.



Once a steady state is attained, the voltage $u_c(t) = U_0 + u(t)$ across the capacitor is has a DC component U_0 and an AC component u(t).

The charge $q_c(t) = C_0 u_c(t)$ of the capacitor also has an AC and DC component: $q_c(t) = C_0 U_0 + C_0 u(t) = Q_0 + q(t)$

ii Linearized electrostatic force

The electrostatic force applied between two electrodes polarised by a voltage U_0 can therefore be written :

$$F_e(t) = \frac{1}{2}\frac{q_c^2(t)}{C_0 d_0} = \frac{1}{2}\frac{(}{}$$

Or $F_e(t) = \frac{1}{2}\frac{C_0 U_0^2}{d_0} + \frac{1}{2}\frac{C_0}{d_0}u^2(t) + \frac{U_0 C_0}{d_0}u(t)$

In practice, the polarisation voltage is set so that $U_0 >> u(t)$

In these conditions $\frac{1}{2}\frac{C_0}{d_0}u^2(t) << \frac{U_0C_0}{d_0}u(t)$, and the electrostatic force becomes:

Constant force due to polarization ↘

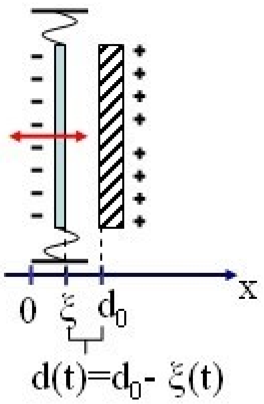
$$F_e(t) \simeq \frac{C_0U_0^2}{2d_0} + \frac{C_0U_0}{d_0}u(t)$$

↖ Linearized alternative force

d) Taking into account the movement of the membrane

i Effect of the membranes movement on the capacity

Until now, the two electrodes were considered immobile. In reality however, the position of the mobile electrode is time dependant $\xi(t)$ due to the electrostatic force. This motion should be taken into account.



The consequence of his motion $\xi(t)$, is that the inter-electrode distance d(t) is time dependant and is written $d(t) = d_0 - \xi(t)$

As the inter-electrode distance is time dependant, the capacity C(t) of the capacitor is also time dependant, and is written:

$$C(t) = \frac{\varepsilon S}{d(t)} = \frac{\varepsilon S}{d_0 - \xi(t)} = \frac{\varepsilon S}{d_0(1 - \frac{\xi(t)}{d_0})}$$

Generally, the motion $\xi(t)$ is a small variation around the resting position $d_0, \xi(t) << d_0$. Thus, the capacity C(t) can be written:

$$C(t) \approx \frac{\varepsilon S}{d_0}(1 + \frac{\xi(t)}{d_0})$$

ii Effect of the membranes displacement on the electrostatic force

The electrostatic force applied to the electrodes is thus:

$$F_e = \frac{1}{2}\frac{q_c^2(t)}{C(t)d(t)} = \frac{1}{2}\,C$$

where $u_c(t) = U_0 + u(t)$, where U_0 is the polarisation voltage, and u(t) is the AC signal, with $u(t) << U_0$

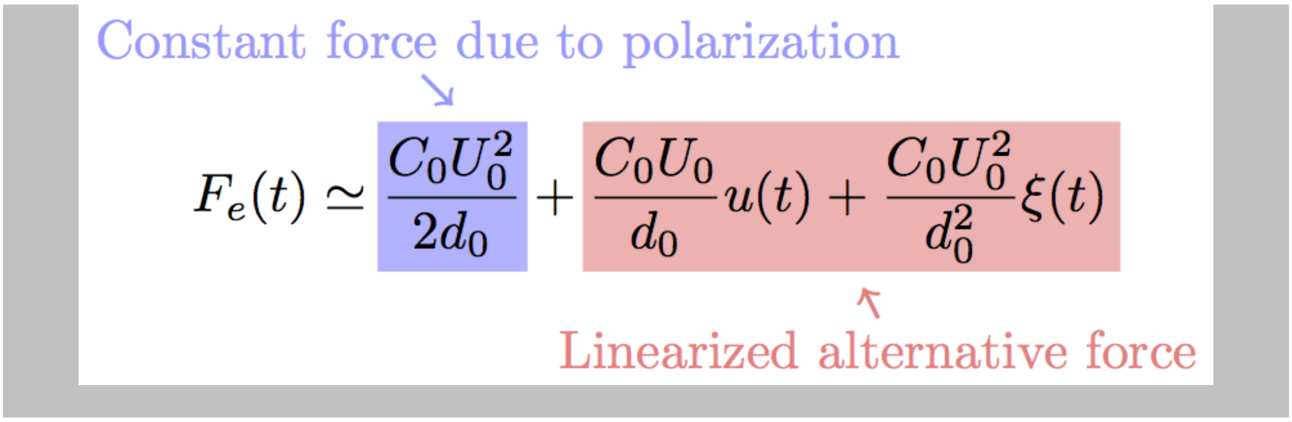
and where $d(t) = d_0 - \xi(t)$ where $\xi(t) << d_0$. In these conditions, the force F_e can be written:

$$F_e = \frac{1}{2}\frac{\varepsilon S(U_0 + u(t))^2}{(d_0)^2(1 - \frac{\xi(t)}{d_0})^2} \approx \frac{1}{2}\frac{\varepsilon S(U_0 + u(t))^2}{d_0}($$

or, after a 1st order development with the variables u and $\xi(u_2(t) << u(t), \xi_2 << \xi)$

$$F_e(t) \approx \frac{1}{2}\frac{C_0 U_0^2}{d_0} + \frac{C_0}{d}$$

iii First equation of electrostatic coupling



By keeping only the AC component, and writing that the velocity v of the membrane is the temporal derivative of the displacement $\xi, v = d\xi/dt$, the electro mechanical coupling equation is written in the harmonic domain $(d/dt = j\,\omega)$

$$F_e \approx \frac{1}{2}\frac{C_0 U_0^2}{d_0} u + \frac{1}{j\omega}$$

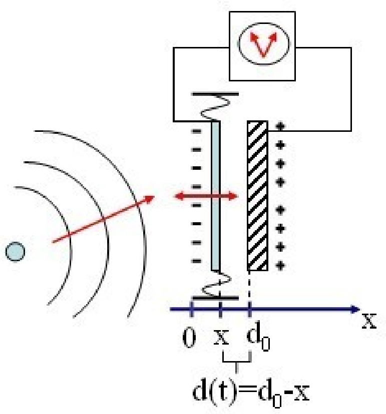
or $F_e \approx k_b u + \frac{k_b^2}{j\,\omega C_0} v$

where $k_b = C_0 U_0 / d_0$ is the **electrostatic coupling factor**

3. Receiver : electro-mechanical coupling

a) Expression of the variable capacity of the transducer

When an acoustic wave vibrates the mobile electrode of an electrostatic transducer, the inter-electrode distance d(t) varies: $d(t) = d_0 - \xi(t)$, where ξ is the displacement of the mobile electrode. The electrical capacity of the transducer varies. The consequence is a voltage variation across the said capacitor.



The displacement ξ modifies the capacity of the transducer as in the following formula $C(t) = \frac{\varepsilon S}{d(t)} = \frac{\varepsilon S}{d_0 - \xi(t)} = \frac{\varepsilon S}{d_0(1 - \frac{\xi(t)}{d_0})}$

Generally, the displacement $\xi(t)$ is tiny compared to the rest position $d_0, \xi(t) << d_0$. Thus the capacity C(t) can be written: $C(t) \approx \frac{\varepsilon S}{d_0}(1 + \frac{\xi(t)}{d_0})$

b) Second equation of electrostatic coupling

The voltage $u_c(t)$ across an electrostatic transducer is the sum of a DC component U_0 and an AC component u(t) : $u_c(t) = U_0 + u(t)$.

The electrical charge $q_c(t) = C(t)u_c(t)$ of an electrostatic transducer is therefore written,

$$q_c(t) = \frac{\varepsilon S}{d_0} U_0 + \frac{\varepsilon S}{d_0} u(t$$

By neglecting the second order terms:

$$q_c(t) \approx C_0 U_0 + C_0 u(t) + \frac{\varepsilon S}{d_0^2} U_0 \xi$$

By conserving only the AC component of this equation, and observing that the current i is the temporal derivative of the charge Q,$i = dQ/dt$ and that the velocity v of the membrane is the temporal derivative of the displacement $v = d\xi/dt$, the temporal derivative of the coupling equation is written

$$i(t) \approx C_0 \frac{d}{dt} u(t) + \frac{\varepsilon S}{d_0^2}$$

In the harmonic domain $(d/dt = j\omega)$, this equation is written

$$i \approx j\omega C_0 u + k_b v$$

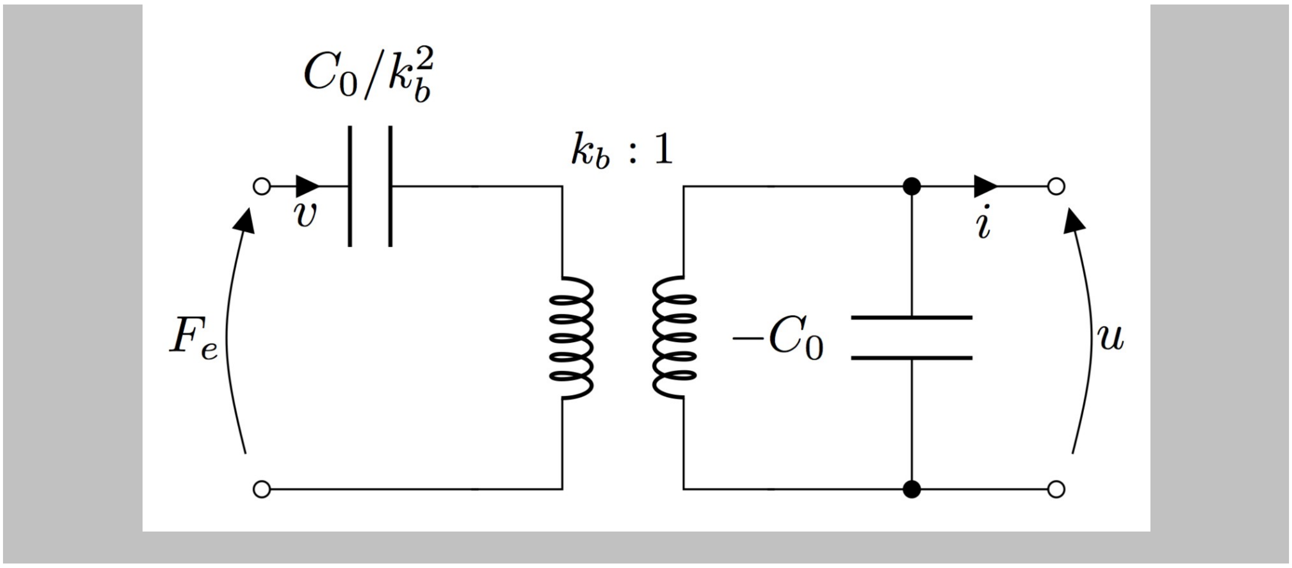
where k_b is the **electrostatic coupling factor**

4. Equivalent electrical representation of the electrostatic coupling

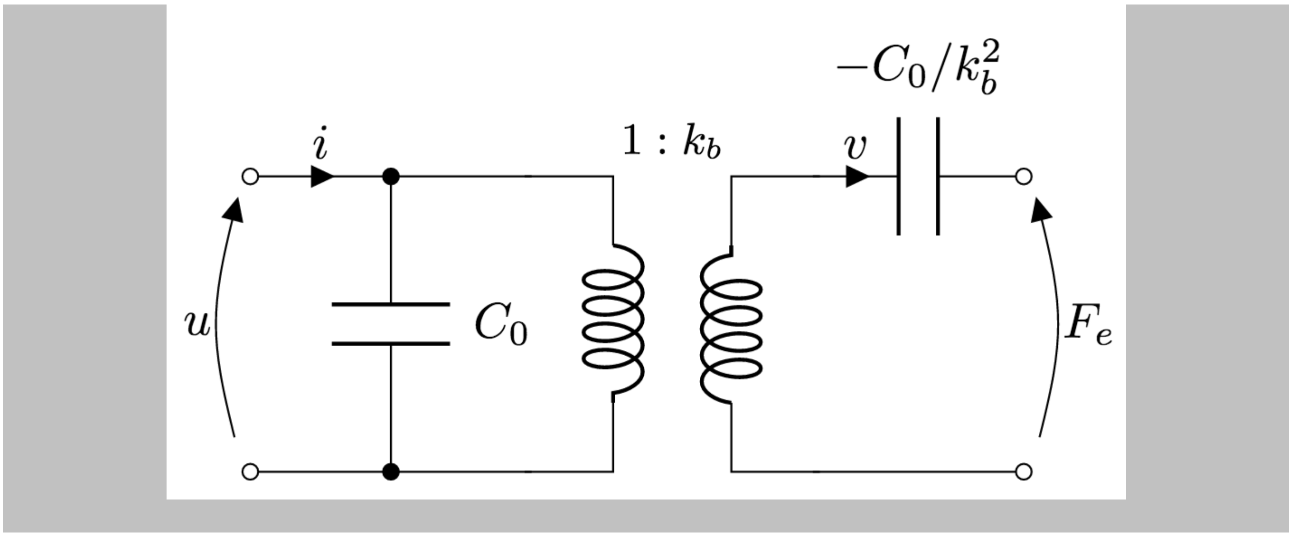
In short, the two coupling equation which describe the electrostatic transduction are the following:

$$\begin{cases} F_e = k_b u + \frac{k_b^2}{j\omega C_O} v \\ i = j\omega C_0 u + k_b v \end{cases} . \quad \mathbf{(25)}$$

These equations can be put in the form of the following equivalent circuit for a receiver:



or in the following form for a generator:



D. Exit test

Exercice 1 : Exercise 1

[Solution n°6 p 18]

The capacity of a capacitor is larger if:

- ☐ the permittivity of the dielectric is weaker
- ☐ the electrode surface is larger
- ☐ the magnetic susceptibility of the dielectric is larger
- ☐ the inter electrode distance is weaker

Exercice 2 : Exercise 2

[Solution n°7 p 18]

The polarisation voltage of an electrostatic generator has a tendancy to:

- ○ Increase the inter electrode distance
- ○ Linearise its behaviour
- ○ Generate an acoustical signal whose frequency is the double of the electrical signal.
- ○ Modify the mediums dielectric constant

Exercice 3 : Exercise 3

[Solution n°8 p 18]

An acoustic wave has the following effect on an electrostatic receiver:

- ☐ a variation of the inter electrode distance
- ☐ a variation of the electrical capacity of the capacitor
- ☐ the generation of a DC voltage at the output of the receiver
- ☐ an amplification of the electrical current flowing through the transducer

Exercice 4 : Exercise 4

[Solution n°9 p 19]

Electrostatic transduction

- ☐ Can be used to make loudspeakers
- ☐ Can be used to make microphones
- ☐ Is intrinsically linear
- ☐ Can easily be miniaturised

Exercice 5 : Exercise 5

[Solution n°10 p 19]

The capacity of a capacitor

- ☐ Is expressed in Farads
- ☐ Is proportional to the inter electrode distance
- ☐ Is proportional to the electrode surface area
- ☐ Is negative when there is a surplus of electrons on the electrodes

Exercice 6 : Exercise 6

[Solution n°11 p 19]

The voltage across a capacitor

- ○ Is always positive
- ○ Is proportional to the stocked charge
- ○ Is inversely proportional to the stocked charge
- ○ Is always proportional to the current flowing through the capacitor

Exercice 7 : Exercise 7

[Solution n°12 p 19]

By neglecting the second order terms, the expression of the electrical charge across the polarised electrostatic receiver:

☐	is time independent
☐	depends on the displacement of the electrode
☐	is inversely proportional to the polarization voltage
☐	includes a term inversely proportional to the square of the distance

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Solution des exercices

> Solution n°1 (exercice p. 3)

◉	The electrical potential across a resistor is directly proportional to the current flowing through it.
○	The resistance of a resistor increases if the voltage drop across it increases.
○	The resistance of a resistor is proportional to the product of the voltage across the resistor and the current flowing through it.
○	The voltage across a resistor is independant of the current flowing through it.

Ohms law is written U=R.I, where R is the resistance, U the voltage across the resistor, and I the current flowing through it.

> Solution n°2 (exercice p. 3)

○	$R_1 + R_2$
○	R_1R_2
◉	$R_1R_2/(R_1+R_2)$
○	$R1/(R_2-R_1)$

> Solution n°3 (exercice p. 3)

○	U_0/ϕ
○	$U_0/2$
◉	$U_0/\sqrt{2}$
○	$U_0\cos(\omega_0 t)$

The RMS value of a variable periodic quantity $x(t)$ is written $x_{eff} = \sqrt{\frac{1}{T}\int_0^T x^2(t)dt}$, where T is the period. Applied to the signal in this question, this formula leads to , after a short calculation, to answer c

> Solution n°4 (exercice p. 4)

- ○ L
- ◉ $jL\omega$
- ○ $1/jL\omega$
- ○ $L\omega$

The electrical impedance is a complex quantity which has an amplitude and phase. The module of the inductors impedance is written: $L\omega$ and the information on the phase is given by the imaginary number j (corresponding to a phase of $\pi/2$)

> Solution n°5 (exercice p. 4)

- ○ an active circuit
- ○ a loss less circuit
- ○ a circuit with a period of 24 hours
- ◉ a resonant circuit

This circuit is composed of passive elements and includes a resistor that is responsible for the electrical losses. Its impedance varies with frequency. The module of this impedance passes by a minimum. This frequency corresponds to a resonance of the current in the circuit.

> Solution n°6 (exercice p. 14)

- [ ] the permittivity of the dielectric is weaker
- [x] the electrode surface is larger
- [ ] the magnetic susceptibility of the dielectric is larger
- [x] the inter electrode distance is weaker

> Solution n°7 (exercice p. 15)

- ○ Increase the inter electrode distance
- ◉ Linearise its behaviour
- ○ Generate an acoustical signal whose frequency is the double of the electrical signal.
- ○ Modify the mediums dielectric constant

> Solution n°8 (exercice p. 15)

- [x] a variation of the inter electrode distance
- [x] a variation of the electrical capacity of the capacitor
- [ ] the generation of a DC voltage at the output of the receiver
- [ ] an amplification of the electrical current flowing through the transducer

> Solution n°9 *(exercice p. 15)*

- [x] Can be used to make loudspeakers
- [x] Can be used to make microphones
- [ ] Is intrinsically linear
- [x] Can easily be miniaturised

> Solution n°10 *(exercice p. 15)*

- [x] Is expressed in Farads
- [ ] Is proportional to the inter electrode distance
- [x] Is proportional to the electrode surface area
- [ ] Is negative when there is a surplus of electrons on the electrodes

> Solution n°11 *(exercice p. 15)*

- ○ Is always positive
- ◉ Is proportional to the stocked charge
- ○ Is inversely proportional to the stocked charge
- ○ Is always proportional to the current flowing through the capacitor

> Solution n°12 *(exercice p. 16)*

- [x] is time independent
- [x] depends on the displacement of the electrode
- [ ] is inversely proportional to the polarization voltage
- [x] includes a term inversely proportional to the square of the distance