Section 2.1 : Electrical systems : Basics review

BRUNO GAZENGEL AND STÉPHANE DURAND

Table des matières

Introduction

I

Objective

The aim of this section is to recall the basic governing laws of electrical circuits.

Prior knowledge needed

Knowledge of basic electrical components (resistor, capacitor, inductor), voltage and current

Knowledge of the complex notation in the harmonic domain (see *section 1.2*[1]).

1 - ../../Grain1.2en/index.html

Test your prior knowledge

II

We recommend that you test your current knowledge. If you do not succeed, you may need to review the basic notions (see *section 1.2*[2]), or the required notions.

Exercice 1 : Test your knowledge

Question 1

What is the unit of the voltage v ?

- ☐ Volt
- ☐ Ampère
- ☐ Coulomb
- ☐ Watt
- ☐ Faraday
- ☐ Ohm
- ☐ Joule

Question 2

What is the unit of current i ?

2 - ../../Grain1.2en/index.html

☐	Volt
☐	Ampère
☐	Coulomb
☐	Watt
☐	Faraday
☐	Ohm
☐	Joule

Question 3

What is the unit of electrical charge q ?

☐	Volt
☐	Ampère
☐	Coulomb
☐	Watt
☐	Faraday
☐	Ohm
☐	Joule

Question 4

When a current flows between two teminals of a conductor:

- ☐ There must be no voltage drop between these points
- ☐ There must be a voltage drop between these points
- ☐ There is a circulation of electrical charges

Question 5

What is the relation between the electrical charge $q(t)$ and the current $i(t)$?

- ☐ $i(t) = q(t)$
- ☐ $i(t) = \frac{dq(t)}{dt}$
- ☐ $q(t) = \frac{di(t)}{dt}$
- ☐ $i(t) = \frac{d^2q(t)}{dt^2}$
- ☐ $q(t) = \frac{d^2i(t)}{dt^2}$
- ☐ $i(t) = \int q(t)dt$
- ☐ $q(t) = \int i(t)dt$

Definitions and conventions

III

A. Two terminal networks

1. The notion of two terminal networks

A two terminal network is an electrical component which has two terminals. Light bulbs, batteries, switches, resistors and motors are examples. We distinguish between two types of two terminal networks:

- Generators: active two terminal neworks,
- Receivers: passive two terminal networks.

Active two terminal network

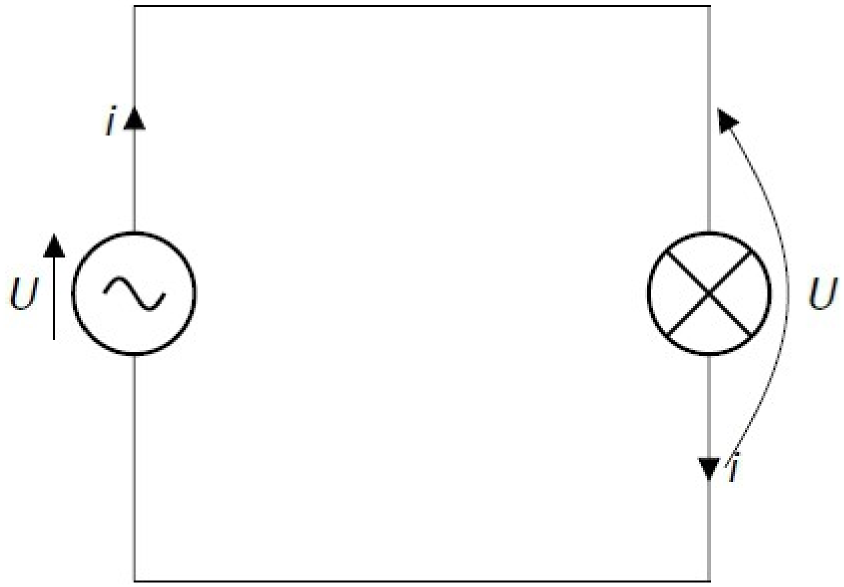
Passive two terminal network

2. Conventions for the orientation of voltage drops and current

The conventional orientation of the voltage and current is illustrated in the following image:

- for generators, the voltage and the current both go in the same direction (left part of the figure),

- for the recievers, the current and voltage go in opposite directions (right part of the figure).



Conventions for the orientation of the voltage and current

B. Quadripoles (two port networks)

1. Notions of quadripoles (two port networks, or four terminal networks)

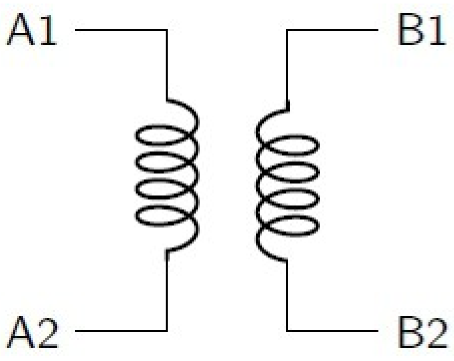
A quadripole is a system with two inputs, each one has two poles. A quadripole allows an energy transfer between two dipoles connected to either input.

The description of a dipole requires the use of four physical quantities:

- The voltage drop across each input,
- the current entering each input.

Exemple : Example

The single phase electrical transformer is a quadripole.

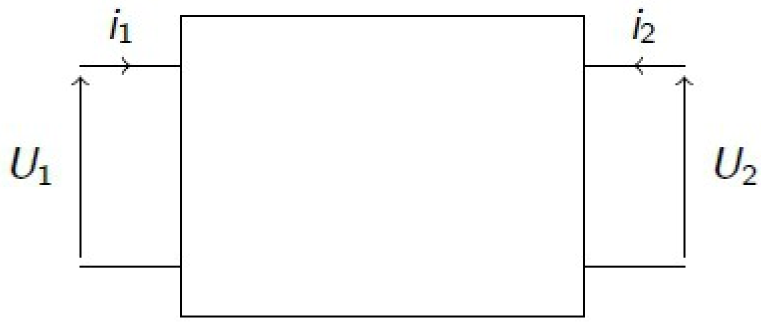


Single phase electrical transformer

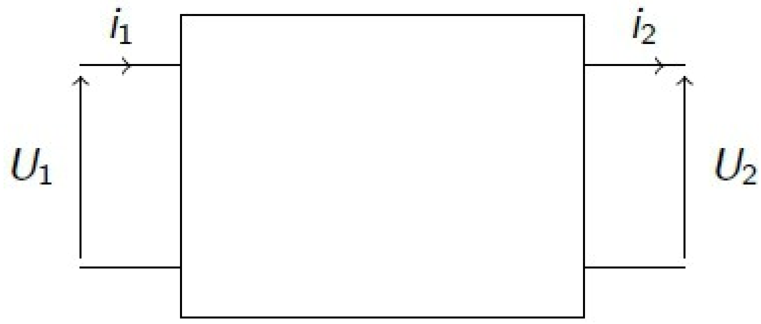
2. Orientation conventions

There are two ways to represent a quadripole:

- symmetrical convention: all the currents enter the quadripole. It is therefore seen as a reciever
  from each side (see reference 9)
- Asymmetrical convention: the quadripole is seen as a system with an ouput and an input. The current enters into the left input, and exits out of the right input. Therefore, the receiver convention is used on the left part, and the generator convention used on the right (see ref 6).



Symmetrical convention

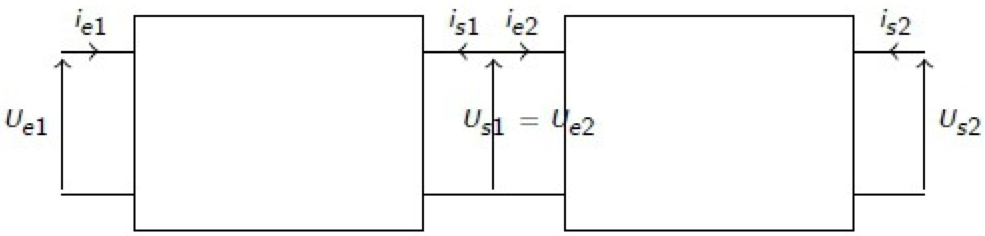


Asymmetrical convention

3. Relations between conventions

The two conventions presented above affect the connection laws between quadripoles.

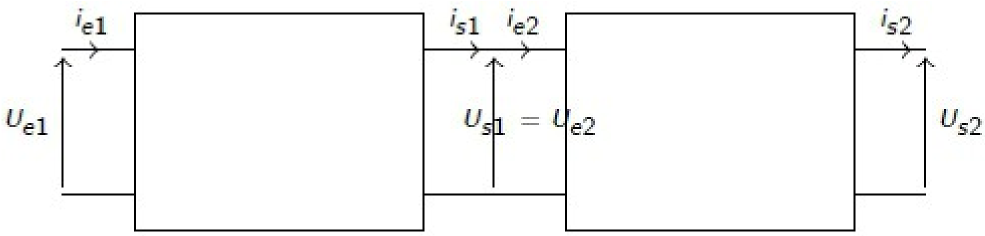
Symetrical convention



Symetrical convention

Under this convention, the sum of all currents in the node between the quadripoles is null $i_{s1} + i_{s2} = 0$ and the voltage drops are equal $U_{s1} = U_{e2}$. The transfer matrix between the two quadripoles is therefore written $\begin{matrix} U_{s1} \\ i_{s1} \end{matrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{matrix} U_{e2} \\ i_{e2} \end{matrix}$

Asymmetrical convention



Asymmetrical convention

Under this convention, the difference between the currents in the node between the two quadripoles is null
$i_{s1} - i_{s2} = 0$ and the voltage drops are equal $U_{s1} = U_{e2}$. The transfer matrix between the two quadripoles is therefore written $\begin{matrix} U_{s1} \\ i_{s1} \end{matrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{matrix} U_{e2} \\ i_{e2} \end{matrix}$.

4. The point of conventions

Symetrical conventions

It is used for the analysis of the energy conversion between different domains (electric ↔ mechanic and mechanic ↔ acoustic).

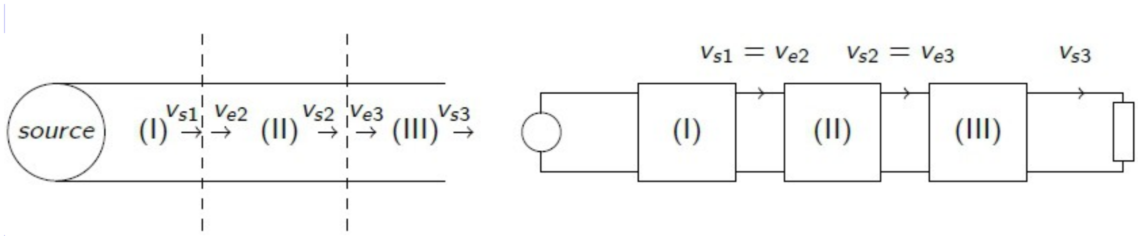
This convention shows that the total power injected into the quadripole is null.

Asymmetrical convention

For mechanical or acoustical systems, for which a source is used before the quadripole, and a receiver after, the asymmetrial convention is preferred to show the continuity of speed or flow.

Exemple : Acoustic transmission line

The schematic on the left shows a real acoustic system, and the one on the right the equivalent electrical circuit at low frequency. It uses three quadripoles under the asymmetrical convention.



Acoustic transmission line

C. Complex notation in the harmonic domain

Reminders

With the hypothesis that the current and voltage are time dependant following a sine law, these variables are written

- for the voltage: $u(t) = u_m \cos(\omega t + \phi_u)$,
- for the current: $i(t) = i_m \cos(\omega t + \phi_i)$.

Using the complex notation (see Section 2.1, reminders: basic notions) these become

- for the voltage: $u(t) = u_m e^{j(\omega t + \phi u)}$,
- for the current: $i(t) = i_m e^{j\omega t + \phi i)}$.

The respective time derivatives are written

- for the voltage: $\dfrac{du(t)}{dt} = j\omega u(t)$,
- for the current: $\dfrac{di(t)}{dt} = j\omega i(t)$.

The respective integrals of the voltage and current can be written

- for the voltage: $\int u(t)dt = \dfrac{u(t)}{j\omega}$,
- for the current: $\int i(t)dt = \dfrac{i(t)}{j\omega}$.

Ohms law

IV

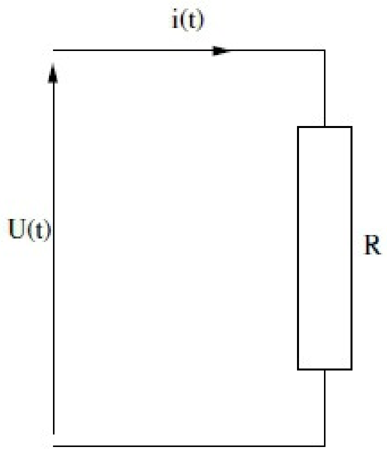
In the case of an electrical resistance, the relation between the macroscopic voltage *u(t)* and current *i(t)* is Ohms law, given by:

$u(t) = Ri(t)$,

where *R* is the resistance in Ω (Ohms).

Complément : Additional resources

In reality, the current that passes through the resistance increases its temperature by the Joule effect. This in turn then modifies the value of the resistance. The resistance is therefore time dependant, and should be written *R(t)*.



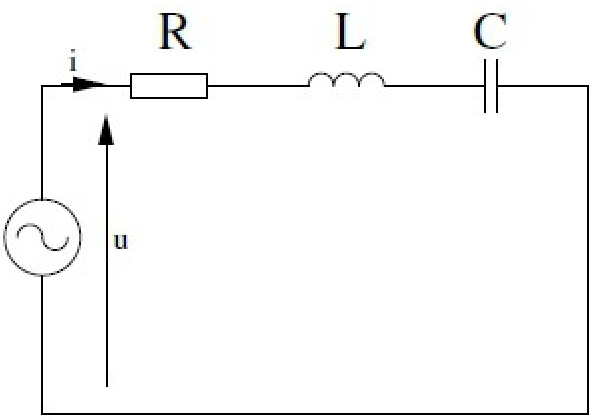
The power dissipated by the resistance is written $P_e(t) = u(t).i(t) = Ri(t)^2 = \frac{u(t)^2}{R}$

RLC circuits

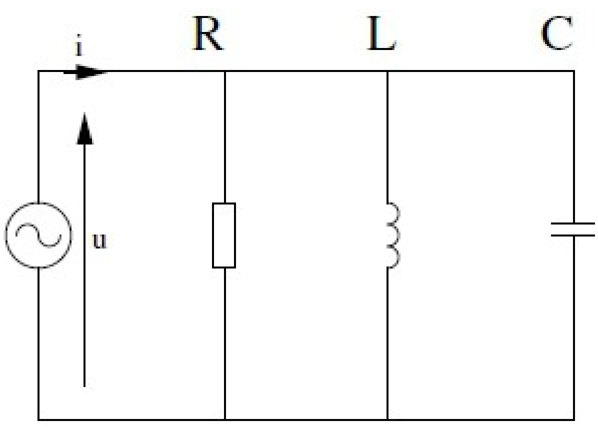
V

A. Contents of an RLC circuit

The RLC circuit is composed of a resistor R, an inductor (a self) L and a capacitor C. These components can be connected in series or in parallel (see the figure below).



Series RLC

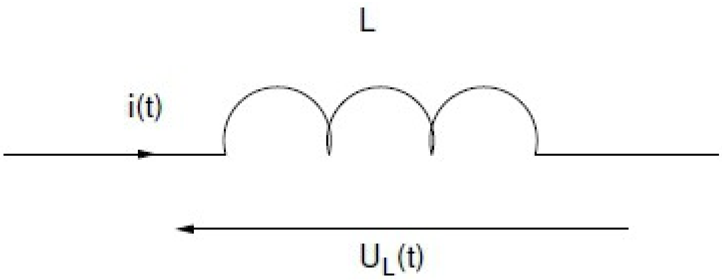


Parallel RLC

B. Time domain

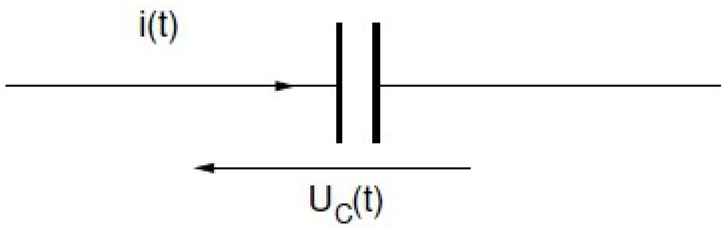
The relations between voltage and current for the classic dipoles (resistor, inductor, capacitor), in the time domain, are written:

- $U_R(t) = Ri(t)$, where i(t) is the current flowing through the resistor and $U_R(t)$ the voltage drop across the resistor.
- $u_L(t) = L\dfrac{di(t)}{dt}$, where i(t) is the current flowing throught the inductor and $u_L(t)$ the voltage drop across the inductor,



Symbol of an inductor

- $i_c(t) = C\dfrac{du_C(t)}{dt}$, where i(t) is the current flowing through the capacitor and u_C (t) the voltage drop across the capacitor.



Symbol of a capacitor

C. Frequency domain

Under the hypothesis that the current and voltage are time dependant following a sine law, the following relations can be written using the complex notation.

- For the resistor $u_R = Ri$, where u_R is the voltage drop across the resistor and i the current flowing through it,
- For the inductance $u_L = j\omega Li$, where u_L is the voltage drop across the inductor and i the current flowing through it,
- For the capacitor $i_C = u_C j\omega C$, or, $u_C = \dfrac{i}{j\omega C}$, where u_C is the voltage drop across the capacitance, and i_C the current flowing through it.

Notions of impedance

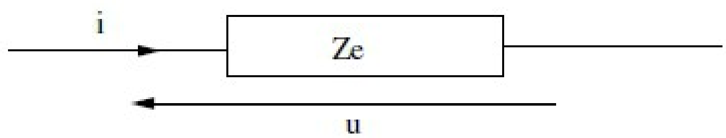
VI

A. The notion of impedance : definition

In the harmonic domain, the behavior of two terminal networks depends on frequency. The impedance of an electrical two terminal network **in the harmonic domain** is:

$$Z_e = \frac{u}{i},$$

where i is the current flowing through the network, and u the voltage drop across it. The impedance therefore measures the reaction of a network to the current i flowing through it.



Impedance of a two terminal network

The impedance is a complex number:

- Its magnitude is the ratio of the current and voltage magnitudes.
- Its phase is a measure of the delay between the voltage and current at a certain frequency.

Complément : Additional resources

Animation: [3]

http://www.animations.physics.unsw.edu.au[4]

3 - http://www.pedagogie.ac-nantes.fr/
4 - http://www.animations.physics.unsw.edu.au/jw/AC.html

B. Admittance versus Impedance

The admittance is defined as the inverse of the impedance. The electrical admittance Y_e is written

$$Y_e = \frac{1}{Z_e}.$$

Méthode : Use of the admittance

In the case of two dual terminal networks connected in parallel, the equivalent admittance Y_{eq} is the sum of the admittances Y_1 and Y_2 of each network:

$$Y_{eq} = Y_1 + Y_2.$$

Méthode : Use of the impedance

In the case of two dual terminal network connected in series, the equivalent impedance Z_{eq} is the sum of the impedances Z_1 and Z_2 of each of the networks:

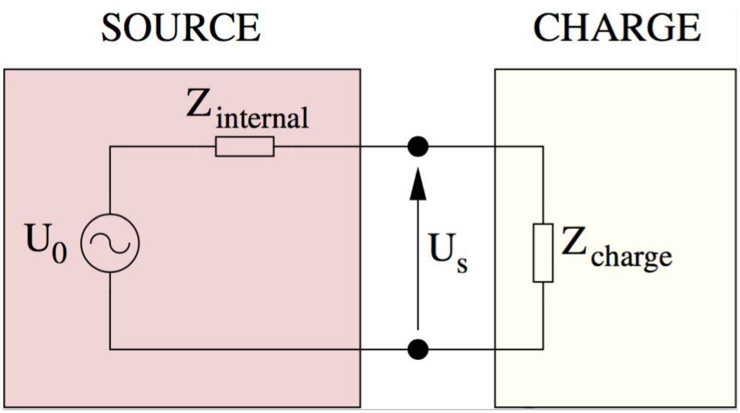
$$Z_{eq} = Z_1 + Z_2.$$

C. Definitions

In the following figure, we note:

- the source impedance : The internal resistance of the generator (voltage or current) which is considered ideal
- the load impedance : The impedance presented by the load.

The internal impedance of the generator can be measured at its output when turned off.



Soure and load impedance

D. Interaction generator-receiver

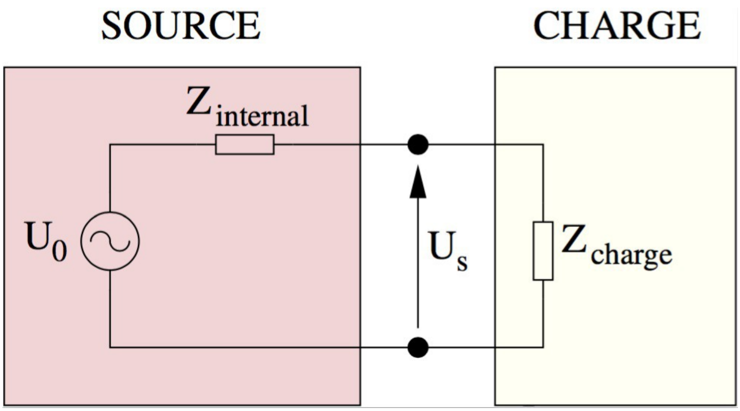
When a generator is connected to a load, there are two way of proceeding.

Optimising the voltage transfer

The ratio $\frac{U_s}{U_0}$ should be maximum. This ratio can be written: $\frac{U_s}{U_0} = \frac{Z_C}{Z_C + Z_i}$. In this case ; $Z_C >> Z_i$ so that $U_s \simeq U_0$.

Optimising the power transfer

For the maximum power to be transmitted from the source to the load, the calculations (detailed *here,*[5] *here*[6] and *here*[7]) show that the relation $Z_i = Z_C^*$, where Z_C^* is the complex conjugate of Z_c



Soure and load impedance

Complément : Additional resources

A transformer can be used to modify the apparent internal impedances of the generator and load to maximise the power transfer.

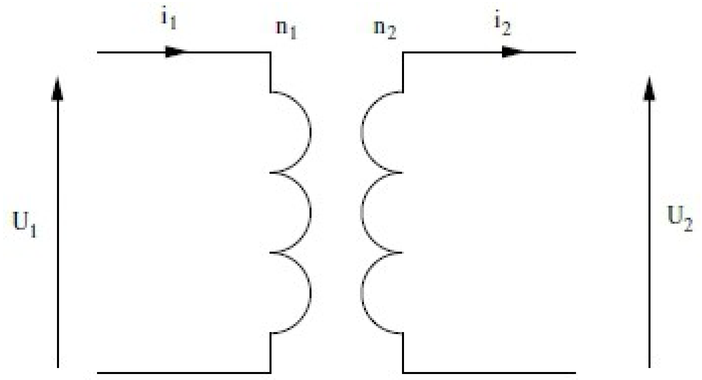
5 - http://uel.unisciel.fr/physique/sinusoi/sinusoi_ch04/co/apprendre_ch4_06.html
6 - http://en.wikipedia.org/wiki/Impedance_matching
7 - http://en.wikipedia.org/wiki/Maximum_power_transfer_theorem

Common quadripoles

VII

A. Coupling equations

Shown below is a virtual lossless transformer. It is used to simulate real electric transformers for coupling effects in electroacoustics.



Ideal transformer

The equations which describe the behavior of an ideal transofrmer are:

$n_1 i_1 = n_2 i_2$,(5)

$\frac{u_1}{n_1} = \frac{u_2}{n_2}$,(6)

where n_1 and n_2 represent the number of primary (subscript 1 on schematic) and secondary (subscript 2 on schematic) windings.

Attention : Caution

The electrical impedance :

$Z_1 = \frac{u_1}{i_1}$

at the input of a transformer depends of the impedance at the output: $Z_2 = \frac{u_2}{i_2}$:

$Z_1 = (\frac{n_1}{n_2})^2 Z_2$.

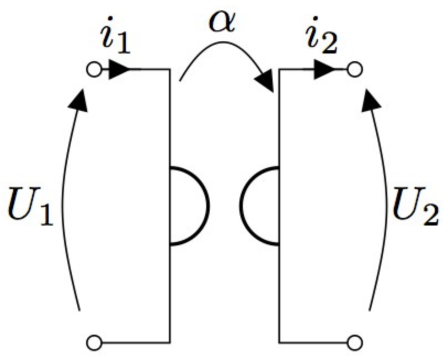
Complément : Additional resources

An illustration of the ideal transformer is given in *this lecture*[8] and *here*[9].

B. The gyrator

An ideal gyrator is a two port network whose input (respectively output) voltage (respectively output) is directly proportional to the output (respectively input) current. The ratio α is usually called the "gyration resistance". In the case of this lecture, we will use the "coupling factor" (for reasons that will become apparent in the following lectures, see *section 3.2*[10]).

In the case of an asymmetrical respresentation, a gyrator is illustrated by:



Gyrator

The relations between current and voltage are written:

$u_1 = \alpha i_2$

$u_2 = \alpha i_1$

where n_1 and n_2 represent the number of primary (subscript 1 on schematic) and secondary (subscript 2 on schematic) windings.

Attention : Caution

The electrical impedance $Z_1 = \frac{u_1}{i_1}$ at the input of the gyrator depends on the admittance at the output $Y_2 = \frac{i_2}{u_2}$: $Z_1 = \alpha^2 Y_2$.

C. To know more

Detailed lectures on quadripoles are given here:

- *http://ressources.univ-*

8 - http://res-nlp.univ-lemans.fr/NLP_C_M14_G02/co/Contenu_13.html
9 - http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/transf.html
10 - ../../Grain3.2en/index.html

lemans.fr/AccesLibre/UM/Pedago/physique/02/cours_elec/quadripo.pdf[11]

- *http://res-nlp.univ-lemans.fr/NLP_C_M14_G02/co/Module_NLP_C_M14_G02.html*[12]
- *http://www.clubeea.org/uploader/mediatheque/Cours-Eln-Muret-Chap6-p217-340.pdf*[13]

11 - http://ressources.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/cours_elec/quadripo.pdf
12 - http://res-nlp.univ-lemans.fr/NLP_C_M14_G02/co/Module_NLP_C_M14_G02.html
13 - http://www.clubeea.org/uploader/mediatheque/Cours-Eln-Muret-Chap6-p217-340.pdf

Electrical circuit analysis

VIII

A. Kirchoff's Law

In a circuit, it is possible to calculate the voltage drops on each dipole, and the current intensity in each branch of the circuit by applying the two Kirchoff laws: the node law and the loop law.

Complément : Additional resources

More information[14]

Millman's theory is a particular form of the node law in which the currents are described by the voltage drops.

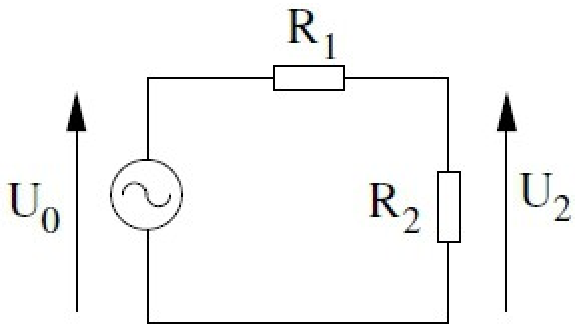
Complément : Additional resources

More information[15]

14 - http://subaru.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/electri/kirchhoff.html
15 - http://ressources.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/cours_elec/millman.html

B. Voltage divider

The voltage divider is a simple electrical circuit that divides the input voltage by a certain value. This value is determined by the ratio between two resistors. The input voltage is applied across the two resistors and the output is taken between the two resistors. The relationship between the output and input is:



Voltage divider

$$U_2 = U_0 \frac{R_2}{R_1 + R_2}$$

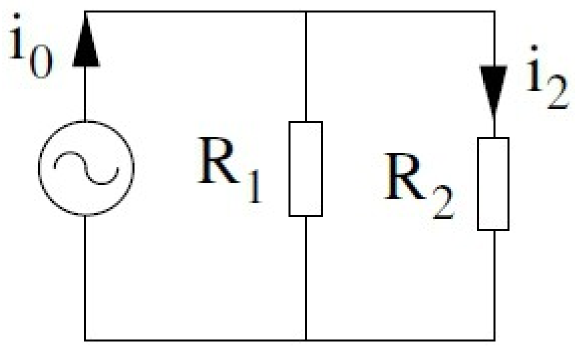
Complément : Additional resources

More information on the voltage divider *here*[16]

C. Current divider

The current divider is an electric circuit that allows the division of the input current by a ratio of resistor values.

A circuit composed of two resistors in parallel can be used to this end. The input current is applied to the circuit, and the output current flows through one of the resistors.



current divider

$$i_2 = i_0 \frac{R_1}{R_1 + R_2}$$

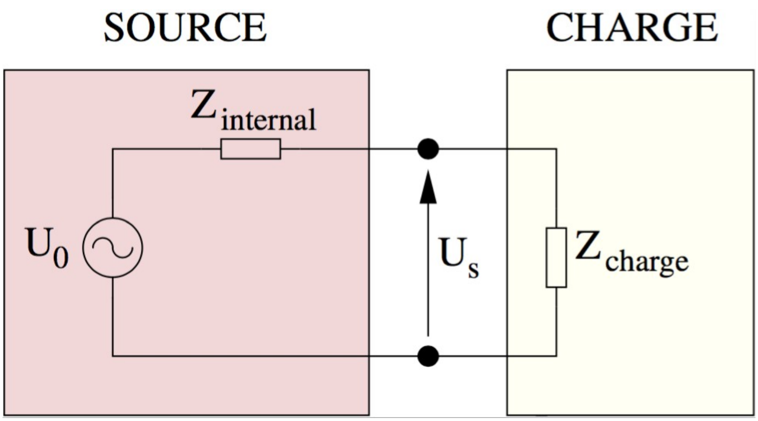
16 - http://subaru.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/electro/potar.html

Complément : Additional resources

More information can be found on current dividers *here*[17]

D. Thevenin generator

The notion of equivalent Thevenin generator implies that we model a real generator with the perfect voltage source U_0 connected in series with a resistor $Z_{internal}$ which represents the internal impedance of the generator. This impedance will influence the output voltage U_s, which will no longer be constant for any value of the load Z_{charge}.



Thevenin generator

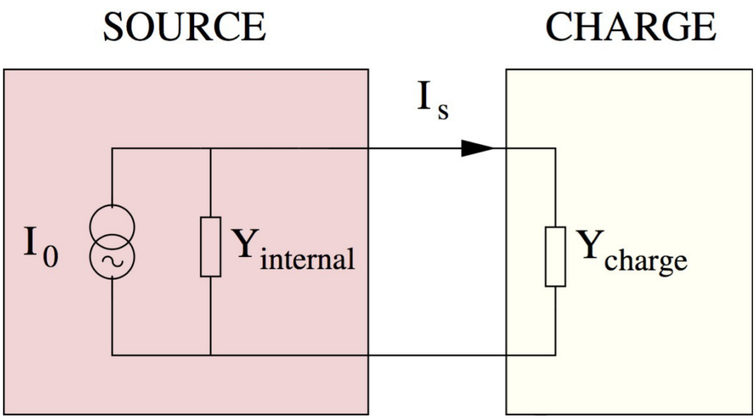
Complément : Additional resources

More information on Thevenin generators *here*[18]

E. Norton generators

The notion of Norton generator allows us to represent an electrical source with an ideal current generator I_0 connected in parallel with an internal admittance $Y_{internal}$. This admittance means the output current I_s will not be constant for every load value Y_{charge}.

17 - http://fr.wikipedia.org/wiki/Diviseur_de_courant
18 - http://subaru.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/electri/thevenin.html



Norton generator

Complément : Additional resources

More information on Norton generators can be found *here*[19]

19 - http://subaru.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/electri/norton.html

Conclusion

IX

A. Summary

- The physical quantities introduced here are the voltage u and the current i.
- The relations between the basic electrical elements are the following (using the complex notation):
  - For a resistor: the voltage and current are proportional $u = Ri$.
  - For an inductance: the voltage follows the time derivative of the current $u = j\omega Li$.
  - For a capacitance: the voltage follows the time integral of the current $u = \dfrac{i}{j\omega C}$.
- The impedance of an electrial system describes the reaction of a system (voltage drop across the system) for a given current that flows through it. The impedance is a complex (magnitude, phase), frequency dependent number.
- The admittance is defined as the inverse of the impedance.
- The most common electroacoustic quadripoles are the ideal transformer and gyrator.

B. Test your knowledge

Exercice 1 : Test your knowledge

Question 1

For an inductance L, the relation between the voltage v and the current i is

Conclusion

- [ ] $u = j\omega Li$
- [ ] $u = Li$
- [ ] $u = \frac{i}{j\omega}$

Question 2

The impedance of a two terminal network

- [ ] Is always a real number
- [ ] Can be a pure imaginary number
- [ ] Is the ratio of current over voltage
- [ ] Is the ratio of voltage over current
- [ ] Does not depend on frequency

Question 3

*Association of two resistances of the same value. For two resistors R **in series**, express the value of the total **impedance***

- ○ $R/2$
- ○ R
- ○ $2R$

Question 4

*Association of two resistances of the same value. For two resistors R **in parallel**, express the value of the total **impedance***

- ○ $R/2$
- ○ R
- ○ $2R$

Question 5

*Association of two impedances of the same value. For two inductors L **in series**,* *express the value of the total* ***impedance***

- $L/2$
- L
- $2L$

Question 6

Association of two impedances of the same value. For two inductors L ***in parallel****, express the value of the total* ***impedance***

- $L/2$
- L
- $2L$

Question 7

Association of two impedances of the same value. For two capacitors C ***in series****, express the value of the total* ***impedance***

- $C/2$
- C
- $2C$

Question 8

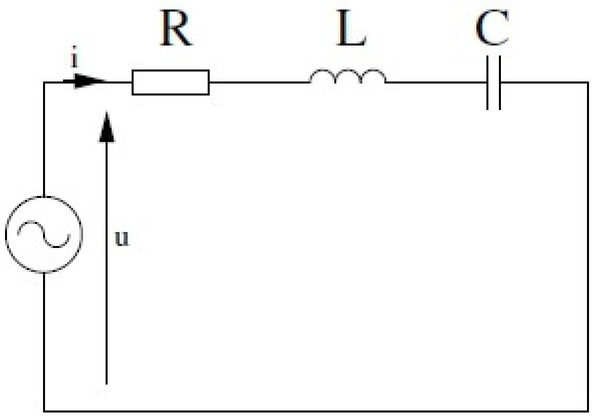
Association of two impedances of the same value. For two capacitors C ***in parallel****, express the value of the total* ***impdance***

- $C/2$
- C
- $2C$

C. Excercise 1: Series RLC circuits

Question

Write the impedance $Z_e = \frac{u}{i}$ of the following system comprised of the elements R, L and C connected in series.

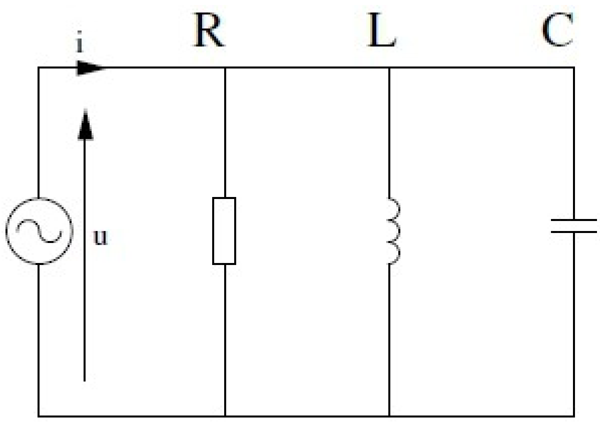


Series RLC circuit

D. Excercise 2: Parallel RLC circuit

Question

Write the expression of the admittance $Y_e = \frac{i}{u}$ of the system comprised of the elements R, L and C connected in parallel. Deduce the expression of the impedance.



Parallel RLC circuit

Bibliography

X

- Jean Jacques Rousseau, Compléments d'électrocinétique, Université du Maine[20] (in French)
- M. Vindevoghel, M. Domon, *Electrocinétique 2 : Régime sinusoïdal permanent*, *cours en ligne Unisciel*[21] (in French)
- J.J. Rousseau, *Physique et simulations numériques*[22] (in French)
- J.J. Rousseau, quadripôles électriques[23] (in French)
- Two port network, *Wikipedia*[24]
- Pierre Muret, Systèmes linéaires à temps continu : quadripôles, filtrage et synthèse des filtres, Université Joseph Fourier, Grenoble[25] (in French)

20 - http://res-nlp.univ-lemans.fr/NLP_C_M14_G02/co/NLP_C_M14_G02_web.html
21 - http://uel.unisciel.fr/physique/sinusoi/sinusoi/co/sinusoi.html
22 - http://ressources.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/electri/rlcserie.html
23 - http://ressources.univ-lemans.fr/AccesLibre/UM/Pedago/physique/02/cours_elec/quadripo.pdf
24 - http://en.wikipedia.org/wiki/Two-port_network
25 - http://www.clubeea.org/uploader/mediatheque/Cours-Eln-Muret-Chap6-p217-340.pdf