Section 1.2 : Basic notions

Pierrick Lotton and Manuel Melon

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Introduction

Objectives

The objectives of this section are

- to recall the basic notions in electronics, mechanics and acoustics needed to understand the UNIT electroacoustic lectures
- to introduce specific electroacoustic notions

Reminder : Basic notions

I

A. Sinusoidal signal

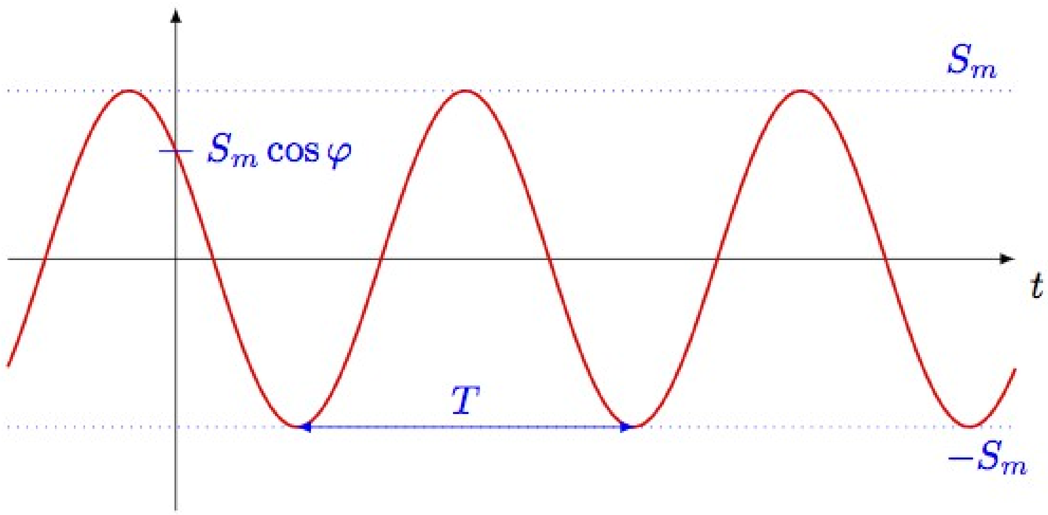
A sine wave is a wave for which the amplitude depends on time, following a sine law. Its mathematical expression $s(t)$ is given by the following relation:

$s(t) = S_m \cos(\omega t + \varphi)$

where :

- S_m represents the maximal magnitude, or the peak value of the sine wave.
- φ represents the phase, in radians. It represents the initial angle of the wave at its origin.
- ω is the angular frequency, and can be calculated with the period T of the wave with the relation: $\omega = \frac{2\pi}{T}$.
- The frequency f of the sine wave is the inverse of the period T: $f = \frac{1}{T} = \frac{\omega}{2\pi}$.

- Illustration:



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B. Real signals

- The sine wave is not often encountered in real life. Acoustic signals are much more complex, containing a large number of different frequencies. These signals can be periodic, and non periodic, and their properties can vary over time, or be steady state.
- We consequently use Fourier analysis for which complex signals are treated like the sum of sine waves. Hence, the electroacoustic systems presented in this lecture will be studied frequency by frequency. This is called a harmonic analysis.

C. Complex numbers

- In harmonic analysis, we use complex numbers : $s(t) = S_m\, e^{i(\omega\, t+\varphi)}$ to represent the signal. The physical signal corresponds to the real part of the complex signal: $\Re[s(t)] = S_m\, \cos(\omega t + \varphi)$.
- To lighten the notation, we omit the temporal dependance and use a complex amplitude s: $s = S_m\, e^{i\varphi}$
- Complex numbers allow simplification of mathematical calculations:
  - The temporal derivation becomes a multiplication:
    $\frac{\partial s(t)}{\partial t} \Rightarrow j\omega \times f(t)$;
  - And the temporal integration becomes a division:
    $\int s(t)\, dt \Rightarrow \frac{f(t)}{j\omega}$.

D. RMS and average values

For time dependant signals, it can be useful to know their characteristics over a certain observation time T_o. The following values are the most commonly used characteristics.

- The maximum and minimum values of the signal $S(t)$ are respectively the highest and lowest points of the signal over a period T_o.
- The average S_{moy} of a signal is given by :

  $$S_{moy} = \frac{1}{T_o} \int_t^{t+T_o} s(t)\, dt.$$

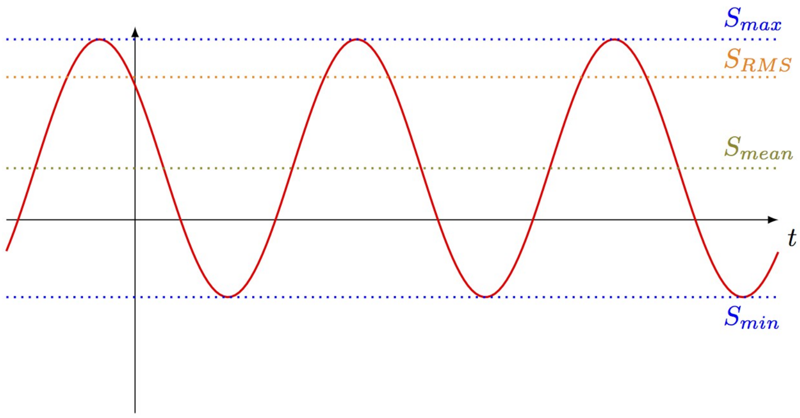
- The RMS value of a signal is given by :

  $$S_{eff} \sqrt{\frac{1}{\mathrm{T}_o} \int_t^{t+\mathrm{T}_o} s^2(t)\, dt}.$$

Remarque : Remark

For periodic signals, the RMS and average value are calculated over a whole number of periods.

- Illustration for a sine wave with a continuous component (DC) equal to S_{moy} :



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Exemple : Example

Calculate the RMS value of the signal $s(t) = S_m \sin(\omega t + \varphi)$.

$$S^2_{eff} = \frac{S^2_m}{T_o} \int_t^{t+T} \sin^2(\omega t + \varphi)\, dt = \frac{S^2_m}{T} \int_0^T \left[\frac{1}{2} - \frac{\cos 2(\omega t + \varphi)}{2}\right] dt.$$

The integral of $\cos 2(\omega t + \varphi)$ is equal to zero, as it is done over two periods.

We therefore have : $S^2_{eff} = \frac{S^2_m}{2T} \int_0^T dt = \frac{S^2_m}{2T} [t]_0^T = \frac{S^2_m}{2}.$

Finally, we obtain : $S_{eff} = \frac{S_m}{\sqrt{2}}.$

E. Electrical quantities

- The voltage is the difference in electric potential between two points. It is symbolised by the letter V , or U and its unit is the Volt (V).
- The intensity of the electrical current measures the quantity of electrical charges that travel through a conductive surface. It is usually denoted i and measured in Amps (A).

F. Mechanical quantities

1. 1D simplification

Even though we live in a multidimensional universe, the majority of the phenomena studied in these lectures will be treated as if unidimensional. For example, the movement of a driver (loudspeaker) membrane is guided in one direction. Thus, we will only interest ourselves with the movement in this direction. Although three dimensional physical quantities are described by vectors we will simplify the notation for one dimension. Hence, the norm of the quantity will be used for the intensity, and a sign (+/-) for its direction.

2. Definition of the quantities

The used mechanical quantities are the following :

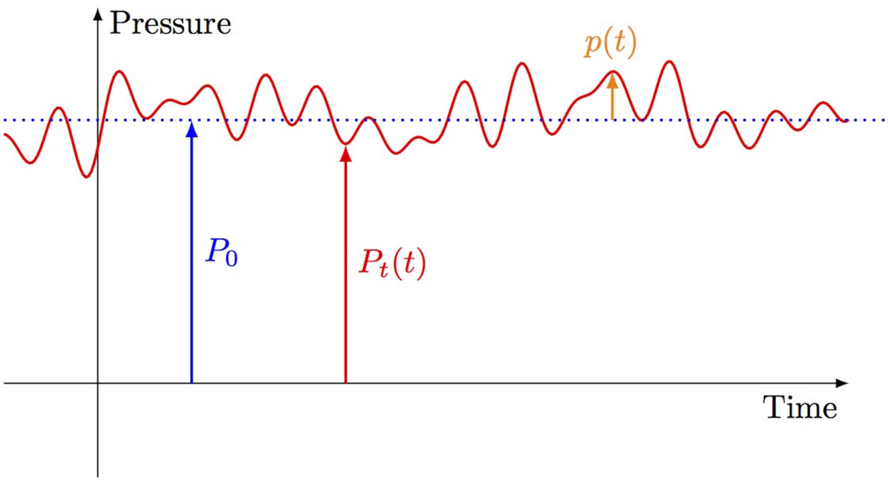
- The position will be noted $\xi(t)$. Its unit is meters (m).
- The instant speed is the time derivative of the position : $v(t) = \frac{d\xi(t)}{dt}$. Its unit is meters per second (m.s^{-1}).
- The acceleration indicates the change of speed over time. The instant acceleration is given by : $a(t) = \frac{dv(t)}{dt}$. Its unit is meters per squared seconds (m.s^{-2}).
- A force is an interaction capable of changing the motion of an object. It is generally represented by the letter $F(t)$ and its unit is the Newton (N).

G. Acoustical quantities

1. Acoustic pressure

In the scope of this lecture, we will study acoustic waves in the ambient air. To characterise an acoustic wave, we use the following quantities : pressure and volume velocity.

- The acoustic pressure corresponds to rapid variations in the atmospheric pressure P_0. Its unit is the Pascal (Pa) and its symbol is $p(t)$. The total pressure $p_t(t)$ at a particular point is given by : $p_t(t) = P_0 + p(t)$. It's a scalar quantity.



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2. Speed and volume velocity

- When an acoustic wave travels through the air, the particles in the fluid oscillate around their rest position at a speed $v(t)$. The volume velocity $w(t)$ measures the fluctuation of this speed trough a surface. Its unit is in cubic meters per second $(m^3.s^{-1})$. The volume velocity is also a scalar quantity.

Remarque : Remark

Be careful not to confuse speed $v(t)$ which corresponds to the vibration speed of the fluid particles, with the speed c of sound which corresponds to the travelling speed of sound waves.

H. The decibel

1. Definition of the decibel

The bel is a logarithmic unit used to express the ratio between two values, G and a reference G_{ref}, of a physical quantity. The decibel is a tenth of a bel. A level L_{dB} is calculated with the following method:

$$L_{dB} = 10 \ \log \frac{G}{G_{ref}} \ (dB).$$

The quantity under study G is, in general, a power. For field quantities g (speed, pressure, force, voltage...) which need to be squared to obtain the power, the level in decibels is given by the following relation:

$$L_{dB} = 20 \log \frac{g}{g_{ref}} \ (dB).$$

2. Uses

The decibel scale is used in numerous physical domains (electronics, acoustics, audio, etc...). As such, this units definition depends on the reference value G_{ref} and can be distinguished by the use of different symbols

- Electronics: dBW, dBV...
- Audio: dBu, dBFs...
- Acoustics: dB A, dB C, dB HL...

Hence, there are a lot of different decibels, which are not comparable. The use of this type of unit requires a sufficient knowledge of the definition of the considered decibel.

3. Gain and attenuation in decibels

The decibel can also be used to explain the gain, or attenuation, of a system. For example, the gain in decibels G_{dB} of an amplifier can be obtained by the ratio of output voltage u_s by the input voltage u_e. The calculation of G_{dB} is done with the following formula:

$$G_{dB} = 20 \log \frac{u_s}{u_e} \text{ (dB)}.$$

4. Acoustic pressure and level

The level of acoustical pressure is used to express the acoustic pressure in decibels. It is calculated using the following equation:

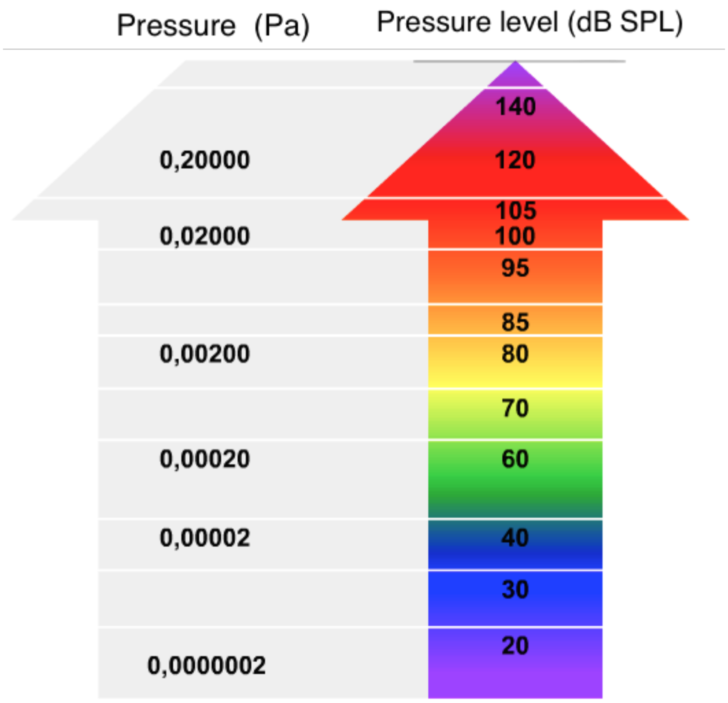
$$L_p = 10 \log \frac{p_{eff}^2}{p_{ref}^2} = 20 \log \frac{p_{eff}}{p_{ref}} \text{ (dB)},$$

where the RMS pressure p_{eff} is given by:

$$p_{eff} = \sqrt{\frac{1}{T_o} \int_t^{t+T_o} p^2(t)\, dt}.$$

The reference pressure $p_{ref} = 20\,\mu$ Pa corresponds to the limit of human hearing at 1000 Hz.

Levels in decibels based on this reference pressure are named dB SPL (with SPL for "Sound Pressure Level").



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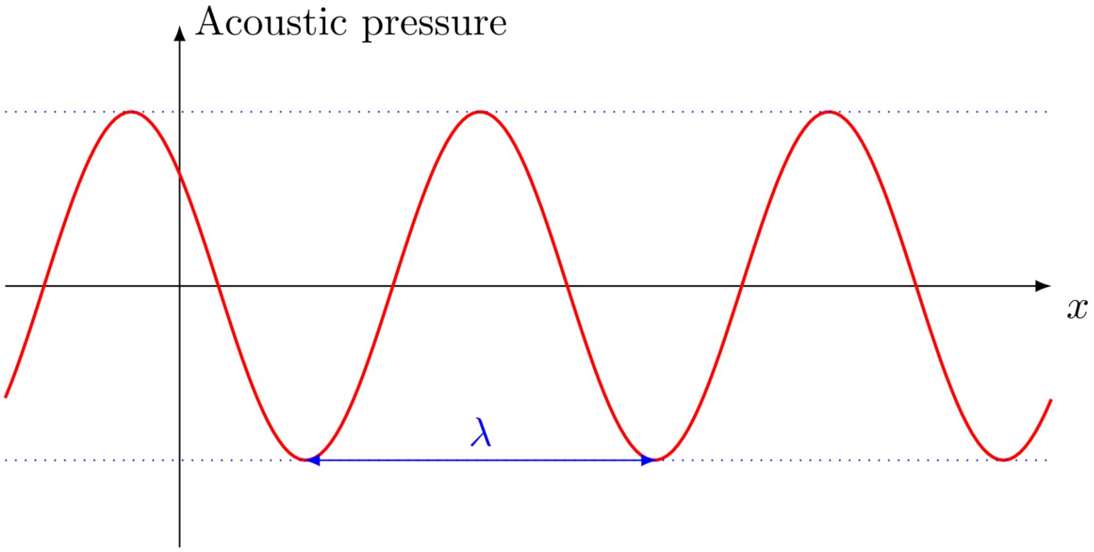
I. Sound speed and wavelength

1. Sound speed

Acoustic waves travel through a perfect medium at a speed that does not depend on frequency. We note this speed "sound speed" and symbolise it with the letter c. If we consider a pure acoustical sine wave of frequency f travelling in one direction of space, for example the axis (Ox), the appearance of the pressure along this axis, at a fixed time t , is that of a sine wave:



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2. Wavelength

The wavelength λ is the spatial equivalent of the time period. It is the smallest distance, over a fixed time t , in which the wave is identically reproduced.

The wavelength also corresponds to the distance travelled by a wave in one period T. As the wave travels at the speed c, we therefore obtain the following relation between the quantities:

$$\lambda = cT = \frac{c}{f}.$$

Remarque : Remark

For audible sounds, the wavelength varies over a large scale : from 17 m at 20 Hz to 17 mm at 20000 Hz. We can see that the wavelength can be very large, or very small, relative to everyday objects.

J. Power

Power is a measure of energy shared between systems per unit of time. The unit for power is the Watt (W). In these lectures, the power is denoted $\mathcal{P}$. The instantaneous power corresponds to the time derivative of the energy E:

$$\mathcal{P}(t) = \frac{dE}{dt}.$$

It is often useful to calculate the average power $\mathcal{P}_{avg}$ over the observation time T_o:

$$\mathcal{P}_{avg} = \frac{1}{T_o}\int_t^{t+T_o} \mathcal{P}(t)\, dt.$$

For the three domains that interest us in this lecture, the instantaneous power is defined by the product of the following quantities:

- Electrical power: $\mathcal{P}(t) = u(t)i(t)$.
- Mechanical power: $\mathcal{P}(t) = F(t)v(t)$.
- Acoustical power: $\mathcal{P}(t) = p(t)w(t)$.

Characteristics of an electroacoustic system

II

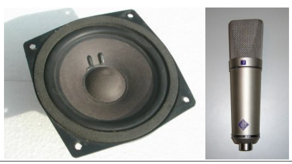
A. Transducers

1. Definition

Définition : Definition

A **transducer** is an object that converts one form of energy to another.

Thermocouples (transforms heat into electricity) or photodiodes (transforms light into electricity) are examples of transducers. In electroacoustics, the most common transducers are microphones, hydrophones, speaker drivers and accelerometers.



2. Types of transducers

The most common electroacoustic transducers can be divided into two groups:

- Sensors, which transform a mechanical or acoustical quantity into an electric one, for example:
  - Microphones;
  - Accelerometers...
- Actuators, which transform an electrical quantity into an acoustical or mechanical one, for example:
  - Loudspeaker drivers;
  - Earphones;
  - Shakers...

B. Sensitivity

1. Definition

The sensitivity of a transducer corresponds to the ratio of the output quantity over the input quantity at a specific frequency. The conditions for which these quantities were measured must be specified (frequency, input level, load,..etc).

2. Sensitivity of a microphone

The sensitivity M of a microphone is given by:

$$M = \frac{u}{p},$$

Where u is the output voltage and p is the acoustic pressure applied to the microphone.

Attention

The ratio must be performed on two quantities of the same mathematical nature: two RMS values, or two maximum values, or two average values,...etc

The sensitivity of a microphone is generally expressed in $\mathrm{mV/Pa}$ or $\mathrm{V/Pa}$, but can also be expressed in decibels by calculating the relative sensitivity:

$$L_M = 20 \log \frac{M}{M_{ref}} \ (\mathrm{dB}),$$

where M_{ref} is the reference sensitivity. This value can differ between manufacturers ($1\ \mathrm{V/Pa}$, $1\mathrm{V}/\mu\mathrm{Bar}$,...).

3. Sensitivity of an actuator

For an **actuator**, the **sensitivity** S is given by:

$$S = \frac{p}{u}$$

where u is the input voltage and p is the acoustic pressure output at a particular position. Its unit is Pascal/Volt $(\mathrm{Pa/V})$.

For loudspeakers drivers, the sensitivity commonly corresponds to an acoustic pressure level on axis at 1 m with the driver mounted in an infinite baffle and subject to a 1W pink noise signal. This signal is tailored to the drivers bandwidth.

Définition : Definition

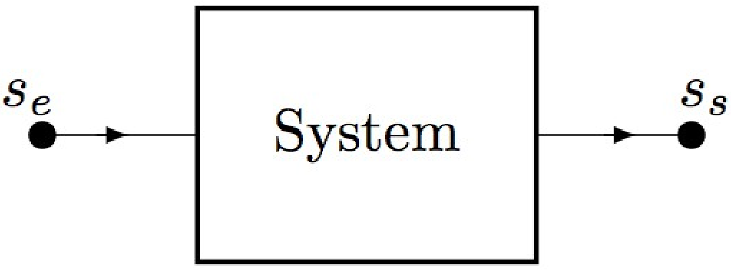
Pink noise is a random signal whose energy is inversly proportional to frequency

C. Frequency response

By applying an input signal to an electroacoustic system, and measuring the output spectrum, we can obtain the frequency response. This response contains information on the amplitude and phase relative to the input signal. However, a large number of manufacturers only give the amplitude curve without the phase. Different kinds of signals can be used to measure the frequency response like, for example, a sinewave with a constant amplitude but varying frequency, an impulse, white noise, pink noise...etc

D. Transfer function

The transfer function H of a linear system is calculated from the complex amplitudes of the input signal
s_e and output signal s_s.



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It is given by the following equation:

$$H = \frac{s_s}{s_e}.$$

The frequency response of audio systems is generally a frequency dependent complex quantity.

E. Bode plot

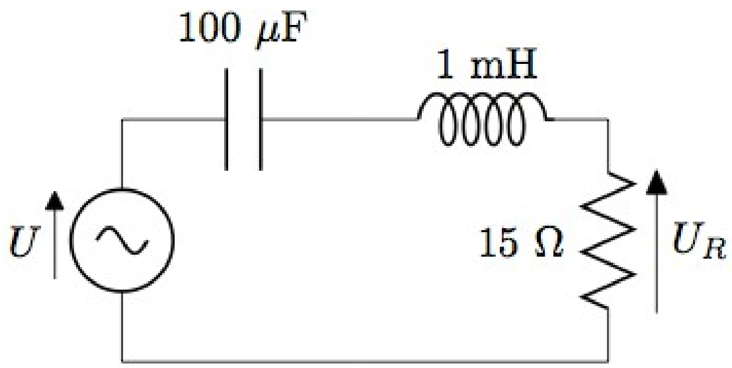
1. Definition

Bode diagrams are used to represent the frequency response or transfer function of a system. They are usually drawn with a logarithmic frequency or angular frequency axis. It contains two graphs:

- The first curve, the magnitude, represents the function : $20\log|H|$, which is given in decibels.
- The second curve, the phase, represents the function $\arg(H)$.

2. Example of a Bode diagram

Let us consider the following electrical circuit:



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The transfer function of the voltage across the resistor, and the input voltage is given by:

$$|H(f)| = \left|\frac{U_R}{U}\right| = \frac{R}{\sqrt{R^2 + \left(L\omega - \frac{A}{C\omega}\right)^2}}$$

Expression of $H(f)$:

$$H(f) = \frac{R}{R + \frac{1}{j\omega C} + j\omega L}$$

The module of this transfer function is :

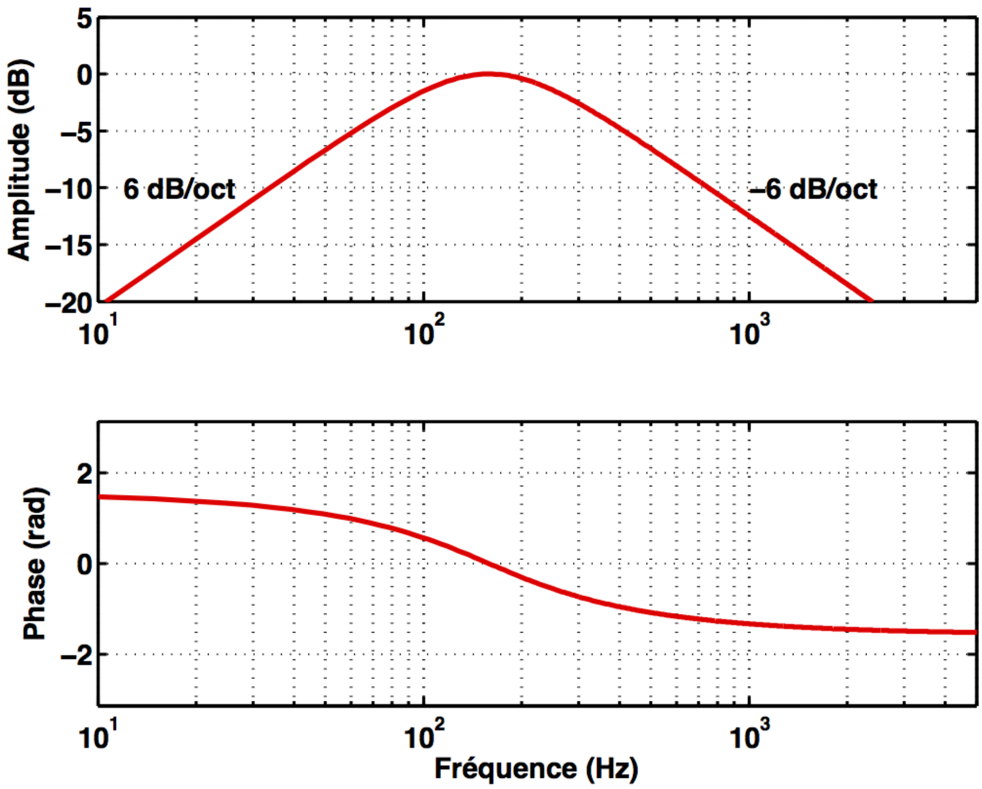
Module :

$$|H(f)| = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

The argument of this transfer function is :

$$Arg[H(f)] = atan\left(\frac{\omega L - \frac{1}{\omega C}}{R}\right)$$

The Bode diagrams of $H(f)$ (in amplitude and phase) are shown below:



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3. Asymptotes

Certain parts of the magnitude curves can be approximated by straight lines. It can be useful to calculate the slopes of these lines. The slopes are generally expressed in dB per octave or in dB per decade.

An octave and a decade are the intervals separating two frequencies. The former corresponds to twice the frequency, and the latter ten times the frequency. Example:

One octave above 1000 Hz is 2000 Hz.

One decade above 1000 Hz is 10000 Hz.

In a range of frequencies where the response is proportional to ω^n, the magnitude difference between 2ω and ω is given by

$$20\log(2\omega)^n - 20\log\omega^n = 20\log\frac{(2\omega)^n}{\omega^n} = 20\log 2^n = n \times 20\log 2 \simeq 6n.$$

So, for example, the Bode magnitude plot of an $n = 2$ system (i.e a system whose transfer function is proportional to ω^2) will have a slope of 12 dB/octave.

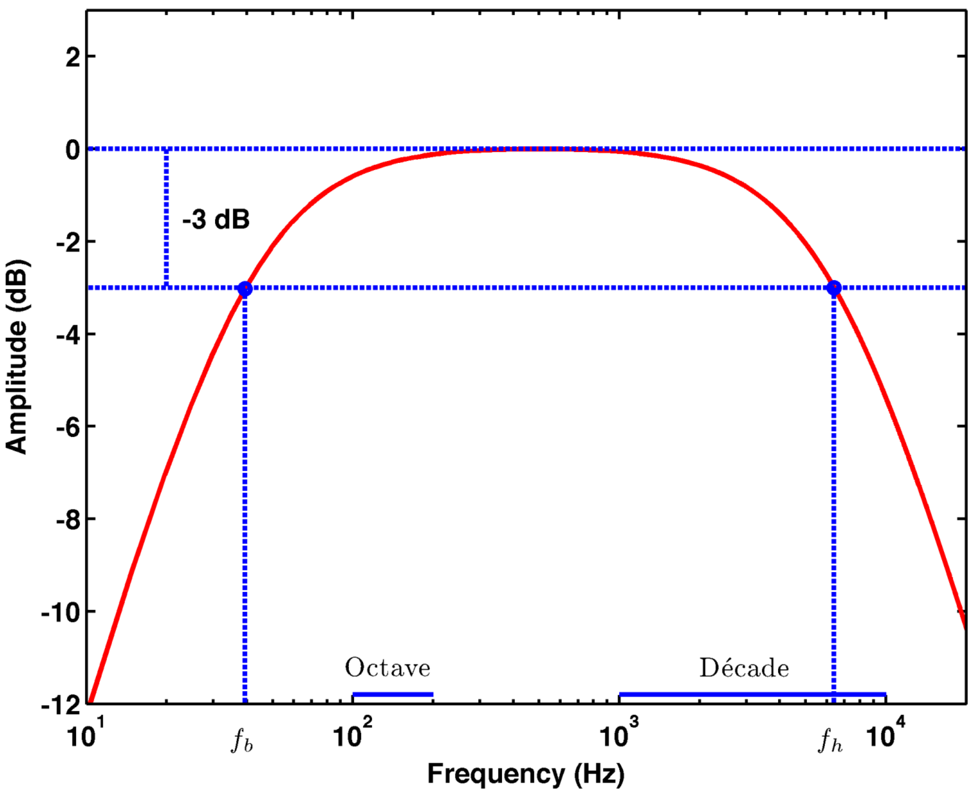
By the same reasoning, the magnitude difference over a decade is proportional to $20n$. Generally, only the value of n is expressed for the frequency interval. For example, $+1$ for a slope of 6 dB/oct, or 20 dB/decade

F. Bandwidth

1. Definition

The bandwidth is a range of frequencies for which the system's response varies between two values. It sits between the lower cut off frequency f_b and high cut off frequency f_h.

The cut off frequencies are chosen ± 3 dB higher/lower than the normal response of the system.



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2. Some remarks

- The bandwidth can be calculated with other values depending on the tolerance needed :
  - Very high, for example : ± 1 dB for accelerometers and instrumentation microphones
  - Low : to avoid cutting the bandwidth into small pieces, like for hearing aids ± 10 dB.
- The tolerance used must always be specified, for example: $70 - 9500$ Hz to ± 1 dB. In some cases, the tolerance my be defined by a standardisation (ISO, AFNOR, AINSI, BSI), and as such may not be mentioned in the technical documentation.
- In general, the measurement conditions must be mentioned, for example : the input voltage and load for a transducer.

3. Bandwidth examples

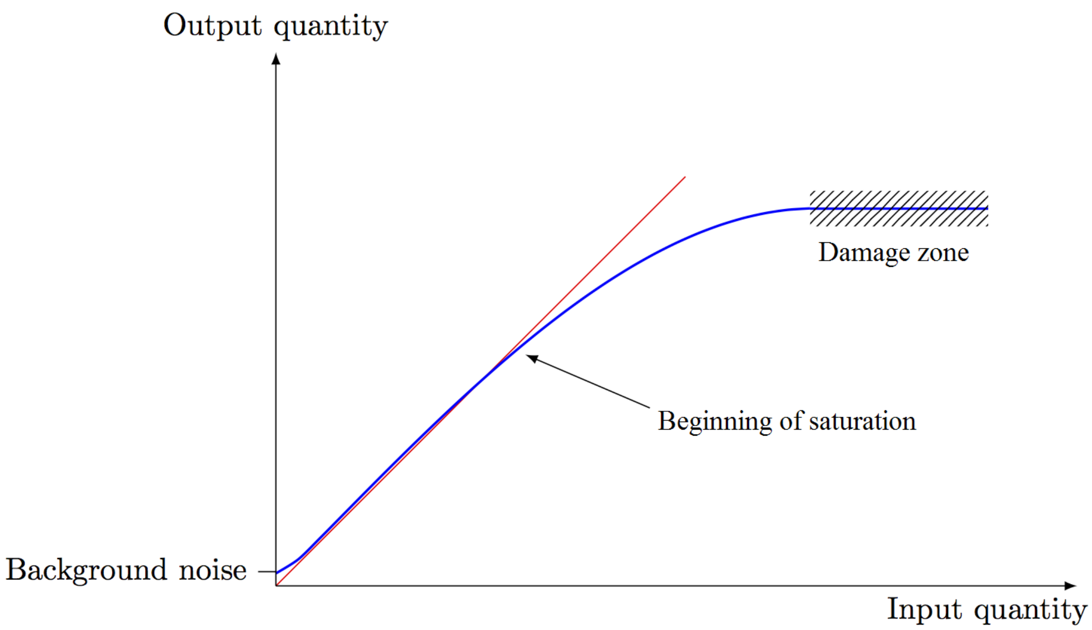
- Frequencies audible to humans : $20 - 20000$ Hz
  - These limits are relative: the highest audible frequency can depend on age, health and the environment. Whereas the frequencies lower than 20 Hz can be heard by the ear at very high levels.
- The voice : 80 % of the information is situated between $\simeq 400 - 4000$ Hz
- Telephone : $\simeq 300 - 3400$ Hz
- Sonic illustration:

- Audible frequency band
- Telephone communications band

G. Dynamic range of a transducer

1. Linear region

The linear region of a real system sits between the noise floor and the physical limits of the system (saturation). This can be seen by representing the ratio between the RMS output and input of a real linear system.



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2. Noise floor

The noise floor is the sum of all the noise sources present in a system. These sources can be considered intrinsic (inherent to the system), or extrinsic (dependant on exterior influences).

- In the **intrinsic noise** category, we find thermal noise (Johnson-Nyquist noise), which is due to the motion of electrons inside an electrical conductor. The noise type is usually white, with a frequency independent magnitude.

However the magnitude is dependant on the impedance value. The formula for the RMS voltage of this noise in a resistor is $\sqrt{4RTK_b\Delta f}$,
where R is the resistance in ohms, T the absolute temperature in Kelvin, K_b Boltzmans constant, and Δf the frequency bandwidth.

The $1/f$ noise due to current fluctuations created by loose contacts, inhomogeneous parts, etc. has a power density which varies aproximatly at $1/f$. It is therefore particularly intrusive at low frequencies.

- Some of the **extrinsic noise** is due to external electromagnetic phenomena like radio waves, RADAR, the mains current, unoptimised switched mode power supplies, etc. Another part can be the product of different mechanical vibrations, such as footsteps, traffic, the drumming fingers of the sound engineer. In the case of acoustical measurements, the background noise created by traffic and equipment situated in and around the building is also picked up along with the signal of interest.

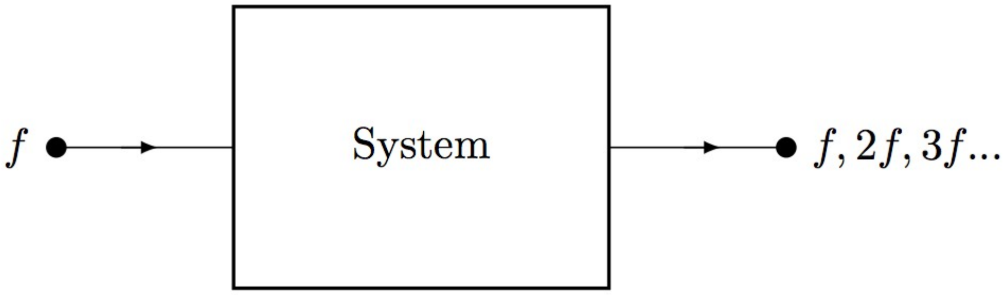
3. Maximum level

When the input attains a certain level, the system will no longer have a linear response due to the physical limits of the system (electronic or mechanical saturation). The output will stay at its maximum (the supply voltage for an amplifier for example), until the input is decreased. If the input is not reduced, then the device can be damaged or destroyed. It is not uncommon to see speakers with the tweeters burnt out due to very high levels.

Hence, it is mandatory to indicate the maximum admissible input levels for a specified quantity (distortion, heat dissipation, etc).

4. Harmonic distortion

A real system will generate harmonics $n \times f$ of its input signals frequency f. These harmonics will be added to the original frequency in the output. This is called harmonic distortion.



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The total harmonic distortion (THD) can be calculated with the following equation :

$$THD = 100 \,.\, \frac{\sqrt{a_2^2 + a_3^2 + a_4^2 + ...a_i^2 ... + a_n^2}}{\sqrt{a_1^2 + a_2^2 + a_3^2 + ...a_i^2 ... + a_n^2}} = 100 \,.\, \frac{\sqrt{\sum_{i=2}^{n} a_i^2}}{\sqrt{\sum_{i=1}^{n} a_i^2}} \ (\%)$$,

where a_1 is the RMS value of the fundamental signal, whereas a_n (for $n > 1$) are the RMS values of the harmonics.

Microphone manufacturers often include a value for the THD for a given pressure level, for example $0,1\ \%$ at 140 dB, which corresponds to a satisfactory performance.

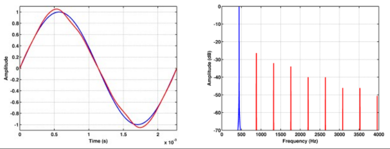
In the case of loudspeakers, the maximum input power rating is given. This corresponds to the level that the loudspeakers can withstand for at least 8 hours without damage. This rating, however, does not give any indication of the distortion, which can rise to very high levels with the highest input power.

When specifying the THD or THD+N (Total Harmonic Distortion plus Noise), the following information should be specified :

- Output power,
- Load,
- Number of harmonics taken into account,
- Input frequency,
- System Gain.

5. Distortion examples

Illustration for a 440 Hz fundamental frequency and a THD of 6 %

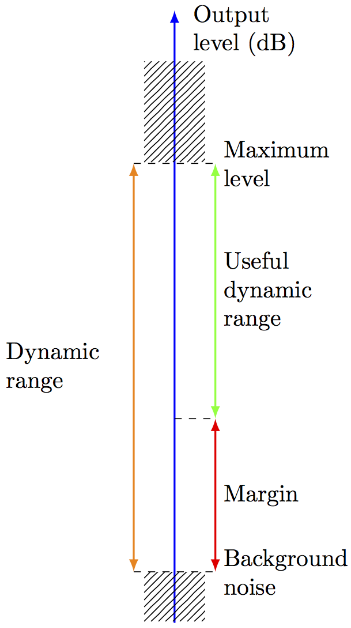


Sonic influence

- Pure sine wave (440 Hz)
- 3% THD
- 6% THD

H. Dynamic range

Useful dynamic range



The useful dynamic range, in decibels, sits between the noise floor and the maximum output level. By reducing the noise floor and increasing the maximal output level, the useful dynamic range can be maximised for a particular system.

Image 1 Pierrick Lotton and Manuel MELON

Exemple : Example

A relatively linear microphone has a noise floor of 24 dB à 1000 Hz and maximum input pressure of 120 dB. What is the dynamic range? What is the useful dynamic range if I wish to a conserve a margin of 20 dB above the noise floor? The dynamic range is $120 - 24 = 96$ dB. The useful dynamic range is therefore $96 - 20 = 76$ dB.

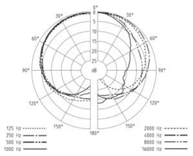
I. Directivity

1. Definition

The directivity represents the response variations over a range of directions. Many transducers are axially symmetric, we therefore study the directivity on a plane with the aforementioned axis being the angular reference $(\theta = 0)$. There are several different types of directivity :

- Omnidirectional: identical response in every direction.
- Unidirectional: highest response in one particular direction.
- Bidirectional: highest response in two opposite directions.

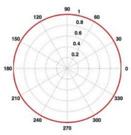
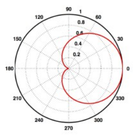
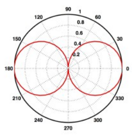
2. Directivity plots

The values on the directivity plot are generally standardised by a particular value (usually the highest value in the response). The quantities can be explained with linear or logarithmic units (dB).	 *Directivity of a Sennheiser microphone for different frequencies*

3. Common directivity patterns

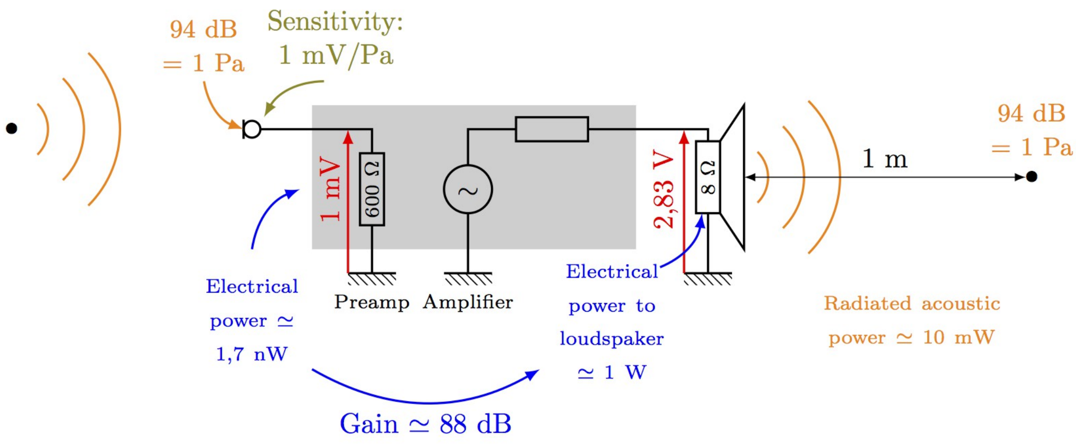
The main directivity patterns are as follows:

- Omnidirectional: identical response in every direction.
- Unidirectional: highest response in one particular direction.
- Bidirectional: highest response in two opposite directions.

Omnidirectional	Unidirectional	Bidirectional
		

J. Orders of magnitude

As an example, the following schematic gives typical values of some quantities



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K. Exit test

Exercice 1 : Exercise 1 - Question 1

The period of a sine wave is:

- ☐ inversely proportional to angular frequency ω
- ☐ inversely proportional to the signal's phase
- ☐ equal to the inverse of the signal's frequency
- ☐ proportional to the signal's peak value

Exercice 2 : Exercise 1 - Question 2

Power :

- ☐ is expressed in Watts
- ☐ is the product of two field quantities
- ☐ is only applicable to electrical systems
- ☐ is always inversely proportional to frequency

Exercice 3 : Exercise 1 - Question 3

Microphone sensitivity corresponds to:

- ☐ the ratio of input pressure to output voltage
- ☐ the ratio of input pressure to background noise
- ☐ the ratio of output voltage to input pressure
- ☐ a unitless quantity

Exercice 4 : Exercise 1 - Question 4

Harmonic distortion:

- ☐ is expressed in volts
- ☐ does not exist in a linear system
- ☐ is due to the presence of odd harmonics
- ☐ should be measured during at least 8 hours

Exercice 5 : Exercise 1 - Question 5

A microphone has a noise floor of 36 dB *à* 1000 Hz *and can detect a maximum of* 140 dB *while staying in its linear region*

What is the useful dynamic range if I wish to keep a margin of 30 dB *above the noise floor?*

- ☐ 74 dB
- ☐ 104 dB
- ☐ 64 dB
- ☐ 110 dB

L. References

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